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PHYSICAL REVIEW.

FATIGUE AND RECOVERY OF THE PHOTO-ELECTRIC CURRENT.[1]

By W. F. Holman.

THE experiments of H. A. Wilson[2] showing that the escape of negative electricity from incandescent platinum disappears when the metal is freed from absorbed hydrogen; and those of Skinner,[3] that in the glow current through rarefied gases hydrogen atoms serve as carriers of negative electricity from metallic cathodes to the gas, suggest the possibility of the absorbed gas playing a part in the escape of negative electricity from a metal subjected to ultra-violet radiation. In fact, the experiments of Wulf[4] indicate that the presence of absorbed hydrogen increases the photo-electric current from platium, in that after the plantinum has been allowed to stand in an atmosphere of hydrogen for some time there is a marked increase of the current over that obtained in air and again a decrease as the metal supposedly loses its charge of hydrogen.

This photo-electric current depends on two principal factors, first that arising from the escape of negative carriers from the metal, and secondly, that from the ionization of the gas by these carriers. Thus the experiments of Stoletow,[5] Lenard,[6] and Varley[7] show that as

[1] Read in part before the joint meeting of the American Physical Society and Section B of the American Association for the Advancement of Science, Ithaca, June, 1906.

[2] Phil. Trans., 202, p. 243, 1903.

[3] Phys. Rev., XXI., p. 1, 1905; Phil. Mag., Nov., 1906.

[4] Ann. d. Physik, 9, p. 946, 1902.

[5] Jour. de Phys., 9, p. 468, 1890.

[6] Ann. d. Physik, 2, p. 359, 1900.

[7] Phil. Trans., 202, 1903.

the gas pressure is reduced indefinitely the photo-electric current drops to a definite minimum value which thereafter remains practically constant for all values of the electric field. This is considered as that arising solely from the carriers escaping from the metal. On the other hand when the discharge takes place in an atmosphere of appreciable density the current may be produced largely by the second factor, the ionization of the gas by the motion of these escaping carriers. For instance, Kreussler[1] found that in air at atmospheric pressure the current increases very rapidly with the electric intensity, if the latter approaches that required to produce a spark discharge.

The present investigation was undertaken therefore with the intention of studying the current arising from the escaping carriers alone, as it is affected by changing the store of hydrogen in the surface of the metal. The results were such as to include a study of certain phases of the so-called *fatigue* and *recovery* of the metal under the conditions of operation.

The plan which suggested itself as most likely to yield conclusive results was based on the discovery of Skinner that as cathode in a glow current, the metal gives off hydrogen, while as anode it absorbs it. In case therefore these conditions can be brought about without otherwise affecting the surface of the metal, this plan furnishes a simple scheme for making a test of the effect of changing the quantity of hydrogen in the metal. As will be seen, however, this condition was attained without question only in one series of experiments.

EXPERIMENTAL ARRANGEMENT.

Fig. 1 presents a diagram of the system used. An electric arc A operated, in parallel with a capacity K, by an induction coil fed by a fifty-volt alternating current served as source of light. This arc was focused by a quartz lens L through a window Q of same material on the cathode E charged from a battery B. The current of negative electricity from E to the anode H, placed opposite, was determined by measuring the rate with which a definite capacity C, connected to the latter, was charged. For this purpose a Thomson quadrant electrometer was employed in the customary manner.

[1] Ann. d. Physik, 6, p. 398, 1901.

The positive pole of the battery and the other terminal of the condenser C were connected to earth. Both the electrometer and the arc system were enclosed in grounded metal cases. Spurious effects were corrected for in all cases by taking observations with the electrode simply screened from the light, other conditions remaining the same.

The electrode chamber is shown in vertical section and plan in Fig. 2.. It was designed for testing several metals successively. These were in form of disks E mounted on the face of a circular glass plate which could be rotated at will about a supporting axis of brass by means of the ground joint D and key K. The electrical connections were made through J, the supporting axis, and

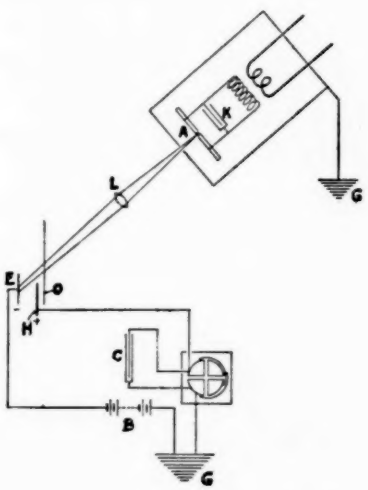

Fig. 1.

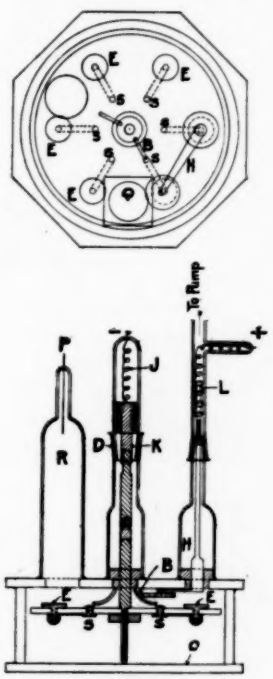

Fig. 2.

the spring brush B to the contact S connected, by tin foil strips placed under the plate, to the cathode E. The connection with the fixed aluminum anode was made through its supporting arm H and the wire L. In order to use the metals E as electrodes with a glow

discharge, a tube R with an additional electrode P was attached to the chamber as indicated. For this purpose a second spring brush similar to B served as connection. The arrangement was such as to insulate all electrodes except the one in action. All fixed joints which could not be fused were sealed with de Khotinsky's laboratory cement. The chamber proved to perfectly air-tight. The electrodes could be renewed by removing the base plate O, which permitted the plate carrying them to be taken out.

In the vacuum system all stop-cocks and ground joints were lubricated with a mixture, recommended by Travers, free of hydrocarbons. Evacuation was produced by a Rapps automatic pump. A McLeod gauge multiplying the pressure about ten thousand times served as pressure indicator. P_2O_5 was used as dryer. A battery of small accumulators served as source of potential.

For an electric arc that found by Varley to be most satisfactory, namely steel electrodes in a slow stream of hydrogen, was used. It proved to be much more constant than other sources tried.

The capacity C (Fig. 1) consisted of an air condenser of tin foil mounted on plane glass plates insulated with quartz.

EXPERIMENTS.

Zinc, carefully polished and cleaned, was tested first. This was mounted in the electrode chamber which was immediately evacuated and left at an indefinitely low pressure in connection with the drying chamber for several hours.

With a gas pressure of .001 mm., a P.D. of about 500 volts[1] and a capacity C (Fig. 1) of about 1,400 cm., the photo-electric current produced in ten seconds the following series of deflections of the electrometer (8.5 scale divisions = one volt). The arc was stopped after each reading and started anew for the next.

TABLE I.

33.4	32.5		32.6
34.4	32.7		31.5
32.9	33.4		30.0
27.0	33.5		30.5
		Mean,	32.0

[1] At the pressures used the photo-electric current proved in all cases wholly independent of the magnitude of the P.D. between the electrodes.

This value is equivalent to 6×10^{-10} amperes. After admitting hydrogen at a pressure of 1.5 mm. a glow current was then sent from the zinc as cathode for about fifteen minutes, and the chamber again evacuated to a pressure less than .001 mm. The photo-electric current observed at this point gave the following successive values:

TABLE II.

20.2	20.2		20.4
20.5	16.1		19.8
20.2	19.5		20.6
20.2	20.6		21.1
		Mean,	20.0

representing a current of 3.8×10^{-10} amperes, which is a marked decrease compared with the fresh metal.

After standing then twelve hours, the following series of observations were taken under the same conditions as before:

TABLE III.

21.3	20.9		21.7
21.5	21.8		22.7
21.3	21.9		22.5
17.7	22.7		22.3
		Mean,	21.5

representing 4.1×10^{-10} amperes. This shows only a comparatively slight increase over the values obtained twelve hours before. Following this, hydrogen was again admitted and the metal used in this case as *anode* during a period of about seven minutes, after which the chamber was evacuated to less than .001 mm. and the following series of values obtained for the photo-electric current.

TABLE IV.

27.5	31.4		32.0
28.0	33.0		32.5
30.1	32.7		32.2
31.5	32.2		
		Mean,	31.2

or a current of about 5.9×10^{-10} amperes which is strikingly close to the first values as given in Table I.

The above results are readily explained in the light of Skinner's experiments if we assume that hydrogen may also serve as carrier of negative electricity in the photo-electric current. With the unused

metal the photo-electric current was 6×10^{-10} amperes. By use as cathode with a glow current the quantity of hydrogen present was undoubtedly considerably reduced. The photo-electric current in this condition was also reduced to about two thirds its original value, 3.8×10^{-10} amperes. Left at rest for several hours the metal indicated practically no change from this last value, but after use as anode in hydrogen — which charges the metal with this gas — the photo-electric current returned to its original magnitude. The same explanation, however, may be given by assuming a migration of electrons with the negative current, if we assume the possibility of appreciably changing their available number in the metal in the same way as the quantity of hydrogen is changed.

A repetition of the above experiments with several different metals brought to light an entirely different set of influences. With electrodes freshly polished the electrode chamber was left at extreme vacuum in connection with the dryer for several hours, after which the photo-electric current was tested as before in an atmosphere of less than .001 mm. pressure. The mean results of a number of tests, in which the individual observations revealed about the same fluctuation in value as those in the previous experiments, are given in Table V. In this series the capacity C was about 2,800 cm., other conditions as before.

TABLE V.

	Photo-electric Current (10^{-10} Amperes).				
	Zinc.	Copper.	Silver.	Aluminium.	Iron.
After using as cathode with glow current in hydrogen.	16	11.6	6.4 0.6	0.6	0.6
After standing 12 hours in hydrogen.	8.8	1.8	14	0	0
After again using as cathode.	41	16		5	4.5
After using as anode in hydrogen.	7	15	6	1.3	1.4
After using as cathode again.	36	16.5	25	2.7	5
After standing in hydrogen several hours.	28	9.3	16	2.5	3.2

A striking feature in these results is the exceptionally large value of the photo-electric current of zinc as compared with the previous tests — the magnitude in one case being as much as seven times as great as the largest values before, and in no case dropping as low as the maximum value previously obtained. In these experiments

it is to be noted that after use as cathode the current is greatest, and after standing in hydrogen or used as anode, it is generally considerably smaller — the different metals being affected in different degrees. The probable explanation is that use as anode or even contact with the gas (possibly not entirely pure) tarnished the surface of the metal and this reduced, as is known, the photo-electric action. In action as cathode with the glow discharge the metal surface in these tests was probably cleaned of tarnish but not sufficiently depleted of its negative carriers to reduce the photo-electric current.

It was thought possible to obtain less complicated results by operating with the glow discharge in argon instead of hydrogen, since according to Skinner use as cathode in this gas serves to deplete the store of hydrogen while use as anode has no appreciable effect. The results obtained are recorded in Table VI. For these experiments, the capacity C was again reduced to 1,400 cm.

TABLE VI.

	Photo-electric Current (10^{-10} Amperes).				
	Zinc.	Copper.	Silver.	Aluminium.	Iron.
After standing *in vacuo*.	1.2	0.5	0.9	1.2	0.2
After use as anode with glow current in argon.	1.2	13	5	0.4	0.7
After use as cathode in argon.	19	11.6	6.3	1.4	0.7
After standing in hydrogen for several hours.	2.4	9	1	0.5	1.2

The effects here are very similar to those recorded in Table V. with the exception that the use of copper and silver as anode with a glow current in argon appears to clean the metal of its tarnish in that it increases its photo-electric current. It should be remarked that in no case did a *visible* tarnish appear when argon was used.

Although the results given in Tables I., II., III., and IV. could not be duplicated, they are so definite that it seems probable that the desired state of depletion of the metal by use as cathode was not reached in the later experiments. Unfortunately further prosecution of the investigation was impossible.

I wish here to express my thanks to Dr. C. A. Skinner of the University of Nebraska for his help and inspiration to me in this work. I am also grateful to Dr. J. E. Almy for much aid and many helpful suggestions.

COEFFICIENT OF LINEAR EXPANSION AT LOW TEMPERATURES.

BY HERBERT G. DORSEY.

IN the present investigation of the coefficient of expansion of substances a direct method of measuring the change in length is used and it is thought that to this end all corrections have been eliminated.

The fundamental idea in the method was first published by Fizeau in 1864 and later developed by Pulfrich in 1893, Tutton, 1898, and Ayers, 1905.

Fizeau[1] first used a tripod (Fig. 1) of steel screws which supported two plates, *D* and *E*. The upper one, *D*, carried a plate of glass, and the lower one served as a table to support the specimen, *F*, to be examined. The upper surface of *F* being polished, monochromatic light passing through *D* and meeting light reflected from *F* produced interference fringes. Any change in the length *FD* would cause a shift of the fringes. The method was devised primarily to examine the expansion of small and rare substances such as diamond. A correction had to be applied for the change in the length of the tripod screws and later he used an alloy of platinum-iridium as being more suitable.

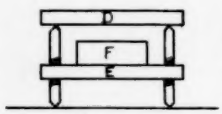

Fig. 1.

Pulfrich[2] used practically the same method but worked out an elaborate correction for the change in wave-length of the light used due to variation in the density of the gas in the space *FD* due to change in temperature.

Tutton[3] modified the method by placing between *E* and *F* a plate of aluminium of such thickness that it would compensate for the change in length of the tripod screws.

[1] Compt. Rendus, Vol. 58, 1864.
[2] Zeit. für Inst., 1893.
[3] Phil. Trans., 191, A, 1898.

Ayers[1] made a great stride towards eliminating corrections by using a hollow cylinder of the material to be examined. By placing it upon a plate of glass and resting another plate above it, interference takes place between light reflected from the top of the bottom plate and the bottom of the top plate the only correction necessary is that due to the change in wave-length. This same method has been mentioned by Scheele[2] and acknowledged by Professor J. S. Shearer.[1] Ayers examined silver and aluminium down to liquid air temperatures, applying the correction by Pulfrich's formula.

Simpson[1] repeated Ayers' work with the same apparatus and exhausted most of the air so as to eliminate at least a part of the last correction. Beginning at this point the problem as presented to the writer was to eliminate entirely this last correction.

The Box.

After several unsuccessful attempts with gaskets of soft rubber and lead, and insulating plugs of rubber and ivory, a cylindrical box was turned from a piece of steel shafting and into the top was ground a piece of thick plate glass as a stopper in a bottle. This was ground in at a small angle so that reflected light would pass out of the field of vision. The joint being slightly lubricated with a mixture of tallow and beeswax will hold a vacuum down to liquid air temperature and when sealed in with shellac will hold up to steam temperature.

Near the bottom a seamless copper tube about 15 cm. long was soldered with silver solder and to the end of this was soldered a longer lead tube. The wires of the resistance thermometer were passed out in the tube and far enough away from the box to be at room temperature were passed through the lead tube, the joint being closed with shellac.

At the bottom of the box was left a tapering lug of steel to conduct the heat from the box slowly. Later a ring of wires of varying lengths was attached. With this arrangement, since the box weighs only about 175 grams, it can be held at any temperature for any desired length of time, and only about a liter of liquid air is necessary for a test of six or eight hours.

[1] Phys. Rev., Vol. XX., Jan., 1905.
[2] Ann. d. Phys. (4), 9, 1902.

THERMOMETER.

Temperatures are measured with a copper resistance thermometer. On a spool of fine silver 2 cm. internal diameter and 0.7 cm. long were wound several layers of No. 36 single silk covered copper wire. To the ends were soldered about 50 cm. of No. 22 double cotton covered wire and to these ends flexible lamp cord of No. 16 equivalent size. To the ends of the flexible cord copper pieces were soldered for terminals. The junctions of the cord and No. 22 are kept wrapped in wool and no thermal effects have been noticed.

The resistance of the thermometer was measured by a Wheatstone bridge, a portable testing set, decade pattern being used in connection with a sensitive reflecting D'Arsonval galvanometer and a Leclanche battery. The same ratio coils, 10 to 1,000, were used both in calibrating the thermometer and in the experimental work. As a further check every coil of the testing set was measured on a bridge correct to 0.1 per cent. and the percentage errors of the testing set computed. The greatest possible error of any combination of coils in the testing set is 0.87 per cent.

In the preliminary calibration dummy leads were run to the coil. Resistance of coil and leads was 67.56 ohms at ice temperature and leads alone 0.182 ohm ; coil in liquid air 11.98 ohms, leads alone 0.166 ohm. Thus the change in resistance of the coil alone was 55.56 ohms and for the leads 0.016 ohm in a change in temperature of 190°. The error, then, which would be introduced by neglecting the change in resistance of the leads would not be over 16 parts in 55,000, or about 0.03 per cent. And since the same lead wires have been used throughout the experiment they are considered a part of the thermometer.

The ice temperature resistance of the thermometer on April 15, 1905, was 67.56 ohms. This was after remaining in distilled water ice thirty minutes, the battery key being left closed all the time. It was noticed later in liquid air tests that with the battery key left closed there was a slight but distinct warming of the coil so that in subsequent measurements the battery key was closed only long enough to produce the galvanometer deflection if any.

On March 14, 1906, resistance in ice with battery key closed only momentarily was 67.53 ohms. Two days later, in fresh, clean snow

resistance was 67.53 ohms. The ice point resistance was taken as 67.53 ohms, and as the two measurements made about a year apart are so nearly alike, a stable condition of the thermometer is indicated.

The low-temperature points were taken from a curve plotted between the resistance of the coil and temperatures of liquid air as

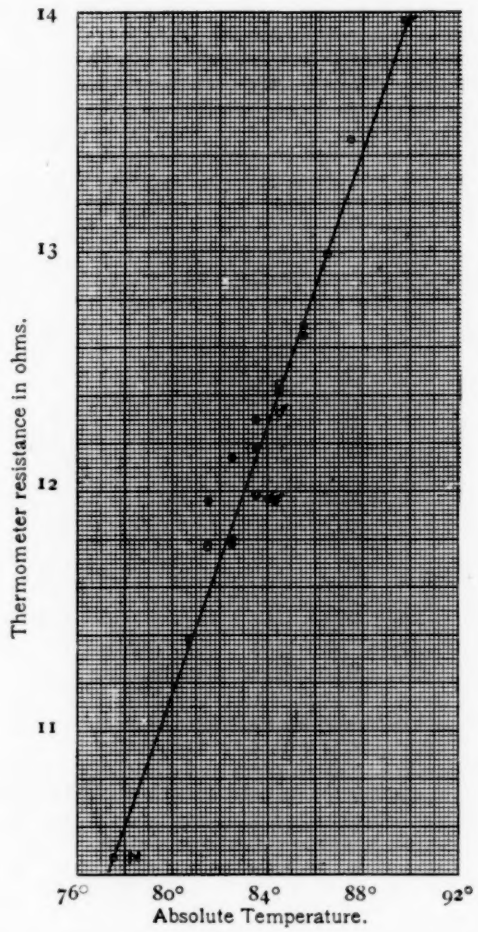

Fig. 2.

indicated by air analysis and Baly's [1] curve and "schwimmers" [2] and liquid nitrogen.

[1] Phil. Mag., Vol. 49, 1900.

[2] Ann. d. Phys., p. 421, 1903.

In Table I. is given the data from which Fig. 2 is plotted. In the curve the air analysis points are marked with a circle and dash.

TABLE I.

Date.	Corrected Resistance.	Temperature.	Method.
April 15, 1905.	11.96	84.3°	Air analysis.
"	11.97	84.0	"
May 26, 1905.	12.33	84.5	"
February 28, 1906.	11.99	83.5	Schwimmer floating.
March 16, 1906.	11.77	81.5	" beginning to float.
"	11.78	81.5	" floating.
"	11.80	82.5	" beginning to float.
"	11.78	82.5	" floating.
"	12.30	83.5	" "
"	12.45	84.5	" beginning to float.
"	12.69	85.5	" " "
"	12.99	86.5	" " "
"	13.47	87.5	" " "
March 19, 1906.	13.96	89.8	Air analysis.
April 6, 1906.	11.96	81.5	Schwimmer floating.
"	12.14	82.5	" "
"	12.18	83.5	" beginning to float.
"	12.43	84.5	" floating.
"	12.65	85.5	" "
"	12.41	84.5	" "
April 14, 1906.	10.47	77.5	Liquid nitrogen.

To determine the steam point the coil was immersed with a Baudin thermometer in a long tube of kerosene and this placed in a jacketed steam bath. Temperatures were taken from the thermometer, the latter being corrected for stem exposed, and for true boiling point determined by barometer. Barometer was corrected for temperature, meniscus and scale reading and difference between values of gravity at Paris and Ithaca. A single careful determination made on March 16, 1906, gave resistance = 96.00 ohms for temperature of 372.12°. On April 11, 1906, under similar conditions six determinations were made, a curve plotted and from this curve the resistance = 95.75 ohms for temperature of 371.00°. On this same date seven determinations were made at room temperature and from a curve plotted the resistance = 73.35 ohms for temperature of 293.00°. These curves are not given as they are of less general interest than the low temperature points.

From the values

Temp. abs.	Res.
372.12°	96.00
371.00°	95.75
293.00°	73.35
273.00°	67.61
83.00°	11.99
77.50°	10.47

an equation was determined, $R = 67.61 + .28904t - .00001913t^2$ (where t is expressed on Centigrade scale). From the equation the resistance was calculated for every 20° from 373° to 93° and a large curve plotted so that temperatures could be read to 0.1°.

Measurements of Lengths.

From the theory of interference in thick plates it is necessary that the surfaces be nearly parallel if the interference bands are to be wide. To realize this condition the specimen, in the form of a hollow cylinder, is cut away slightly at each end leaving three small projections. These are then adjusted by fine grinding and measuring until planes passed through them at both ends will form an angle of about 10 or 20 seconds of arc.

The measurements were made with micrometer calipers which were compared at 22° C. on a Rogers and Ballou dividing engine with a W. A. Rogers bar meter and yard, After applying bar and temperature corrections according to Rogers' specifications the micrometer screw was found to give average readings 0.13 per cent. too small. Adding this amount to the specimen measurements gives their lengths in terms of the Metre des Archives.

1.	2.	3.
1.1258	1.1243	1.1258
1.1254	1.1245	1.1252
1.1255	1.1242	1.1256
1.1259	1.1240	1.1255
1.1254	1.1240	1.1259
1.1256	1.1242	1.1254
		Average = 1.1251
		Subtract .0005 (zero error)
		1.1246
		add 0.13 per cent. .0015
		length = 1.1261 cm.

Temperature = 22° C. = 295 abs.

At least five measurements of each of the three projections were made for each specimen and the average of the three sets taken. A sample of the measurements is given on previous page which is for gold as first tested.

ARRANGEMENT OF APPARATUS.

The general arrangement of the apparatus is shown in Fig. 3. Light from a mercury vapor lamp at *A* passes through a slit and is made parallel by the lens *B*. Passing through the prism *C* set

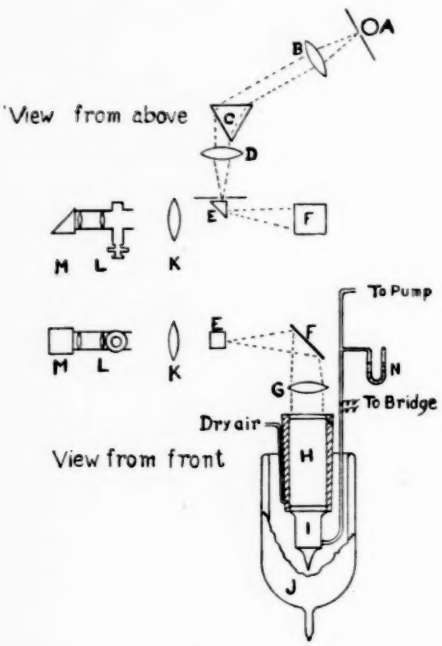

Fig. 3.

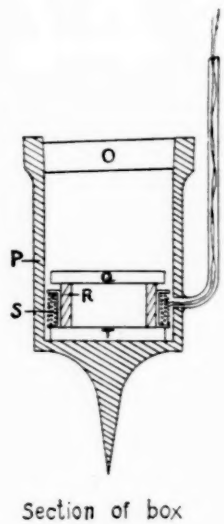

Fig. 3a.

for minimum deviation, the lens *D* focusses it on a very small slit and by the right angle prism *E* it is reflected to the mirror *F* silvered on its front surface. Thus far the light paths are horizontal, but the mirror *F* reflects it downwards and the lens *G* again renders it parallel. After passing into the box *I*, producing interference bands it is brought to a focus at *E*, just at the edge of the prism. The lens *K* produces a real image of the fringes at the plane of the cross wires in the micrometer eye-piece *L* and another right angle prism *M* reflects the light to the eye.

In the section of the box the interference fringes are formed at the lower surface of the glass wedge *Q*. On this surface two fine crossed lines were scratched which serve as a reference point. *T* is a disc of black glass polished on its upper surface only upon which rests the specimen *R*. *S* is the resistance thermometer and *O* is the glass window of the box.

The box is held by friction in the wooden tube *H*. To keep the window *O* from frosting over dry air is passed in at the bottom of the tube *H* and out at the top, the upper end being closed by a glass plate. The best and most convenient source of dry air yet found is obtained from the slowly evaporating liquid air in a Dewar bulb.

The cylindrical Dewar bulb *J* containing liquid air for cooling is raised or lowered by a hand wheel on the axle of which is wound a strap carrying a balanced platform which carries the bulb.

The box is exhausted by a mercury pump and a manometer *N* is placed so as to be easily seen. The Wheatstone bridge is placed on the table in front of the observer so that adjustments can be easily made with the left hand, leaving the right to control temperature. The scale of the galvanometer is in line with *M*, so that the spot of light can be observed without turning the head.

Method of Observation.

The specimen being sealed in the box and adjusted to give good interference fringes, the box is exhausted to something less than a mm. pressure and the Wheatstone bridge set for some given temperature. The box is then warmed or cooled until the spot of light remains stationary at zero and the fringe system ceases to move. Usually no dark band will be on the reference mark so its distance is measured as a fraction of the distance between two dark bands. The box is then rapidly cooled (or warmed) and the bridge set for the next temperature. Meanwhile the fringes are counted as they pass the reference mark by pushing the handle of a counting machine. The box will soon have reached the desired temperature and is held there until the fringe system stops moving. This occurs nearly the same time that the galvanometer comes to zero. For the amount of matter in the specimen is so small, only a few

grams, and it is so near the thermometer, being surrounded by it, that it very quickly comes to the same temperature. The entire time occupied in covering a 20° interval is usually 20 to 30 minutes. After all motion has ceased the fringe fraction to be added

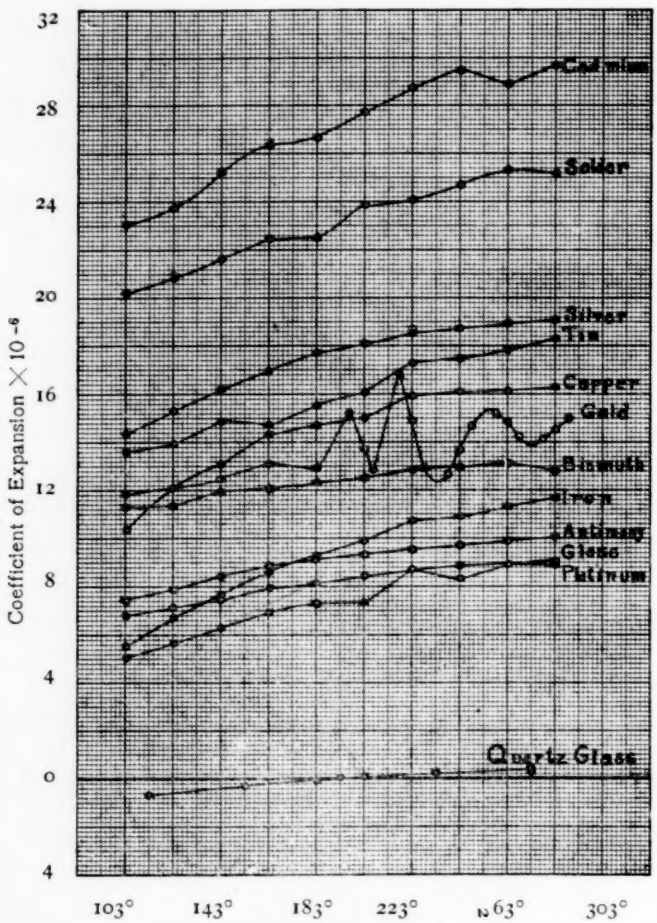

Fig. 4.

is measured and that with the number on the counter and the resistance is recorded.

This process is repeated by 20° intervals down to 93° and then again as the box is warmed to room temperature. Thus the change in length is measured both in cooling and warming the specimen

and the average of all the measurements for any given interval gives the amount used in calculating the coefficient of expansion for that interval.

Since the light passes the length of the specimen twice a whole fringe counted means an actual change in length of one half the wave-length of light used so that the coefficient is $e = n\lambda/2dt$ where n is the number of dark spaces counted, λ the wave-length, d the length of the specimen, and t the temperature interval, usually 20°. The green line of mercury was used, the wave-length of which is $54{,}607 \times 10^{9}$ cm., according to Fabry and Perot. The coefficient thus determined is taken as the average for the temperature interval and is plotted at the middle of the interval in Fig. 4.

Table II. gives the middle temperature of the interval, that at which the coefficient is plotted, the number of fringes counted and their average, the calculated coefficients, average coefficient between 293° and 93°, and description of material.

TABLE II.

Antimony. Cast.

From Department of Chemistry, marked chemically pure. Length = 1.3409 cm. Average width fringes = 180 divisions of micrometer eye-piece head. Density = 6.88. Average coefficient from 293° to 93° = $8{,}170 \times 10^{-8}$.

As shown in the table, the specimen actually expanded more on warming than it contracted in cooling.

Temp.	283°	263°	243°	223°	203°	183°	163°	143°	123°	103°
Cooling.	10.04	8.59	8.75	8.66	8.56	8.17	7.82	7.05	6.41	6.16
Warming.	9.78	10.66	9.84	9.77	9.31	9.28	9.22	8.61	8.22	8.00
Cooling.	10.13	10.06	9.97	9.92	9.65	9.19	8.89	8.57	8.18	7.37
Warming.						9.25	8.91	8.84	8.31	7.89
Cooling.	9.86	9.57	9.63	9.30	9.16					
Average.	9.95	9.72	9.55	9.41	9.17	8.97	8.71	8.27	7.78	7.35
$e \times 10^{-8}$.	1,009	988	970	956	933	913	886	841	791	747

Cadmium. Cast.

From Eimer and Amend, marked pure. Length = 1.1659 cm. Average width fringes = 200. Density = 8.62. Average coefficient = $2{,}692 \times 10^{-8}$. Coefficient corrected for change in length at lower temperatures

	25.39	24.28	25.16	24.78	23.33	22.63	22.48	21.48	20.38	19.67
	25.56	24.73	25.17	24.78	23.78	22.61	22.38	21.50	20.23	19.33
	25.32	24.94	25.20	24.19	23.65	22.77	22.52	21.57	20.00	19.64
	25.16	24.77	25.18	24.37	23.51	22.71				
						22.73				
Average.	25.38	24.68	25.18	24.53	23.57	22.69	22.46	21.52	20.20	19.55
$e \times 10^{-8}$.	2,970	2,890	2,948	2,876	2,768	2,664	2,638	2,527	2,379	2,302

Bismuth. Cast.

From Department of Chemistry, marked Free from Arsenic. Length = 1.3409 cm. Average width fringes = 170. Density = 9.84. Average coefficient = $1,224 \times 10^{-8}$.

Temp.	283°	263°	243°	223°	203°	183°	163°	143°	123°	103°
	14.50	14.73	14.58	14.49	14.07	13.78	13.49	13.37	12.65	12.69
	14.60	14.76	14.49	14.40	14.14	13.88	13.53	13.56	12.93	12.71
	14.26	14.79	14.73	14.55	14.03					
	14.24									
Average.	14.40	14.76	14.60	14.48	14.08	13.83	13.51	13.46	12.79	12.70
$e \times 10^{8}$.	1,280	1,311	1,298	1,287	1,251	1,230	1,201	1,196	1,136	1,129

Copper. Commercial seamless tubing.

Analyzed by Mr. C. G. Schluederberg, fellow in chemistry, and found to be 99.9 per cent. copper. Length = 0.9712 cm. Average width fringes = 190. Density = 8.91. Average coefficient = $1,102 \times 10^{-8}$.

	11.80	11.34	11.50	11.00	10.59	10.44	10.31	9.49	8.59	7.69
	12.05	11.47	11.34	11.61	10.80	10.69	10.24	9.44	8.40	7.35
	11.50	11.40	11.94	11.16	10.65	10.51	10.00	9.16	8.72	7.26
	11.40	11.14	11.55	11.51		10.25	10.19	9.25	8.57	7.24
	11.63	11.69	11.09	11.55		10.31	10.00	9.25	8.70	7.43
	11.72	11.79	11.81	11.28			9.98	9.18	8.55	7.37
	11.30		10.90				10.77			
Average.	11.63	11.47	11.45	11.35	10.68	10.44	10.21	9.29	8.59	7.39
$e \times 10^{8}$.	1,635	1,613	1,610	1,595	1,500	1,468	1,437	1.305	1.206	1,039

Glass. Crown tubing.

Length = 0.9673 cm. Average width fringes = 230. Density = 2.45. Average coefficient = 736×10^{-8}.

	6.37	6.40	5.74	6.26	5.00	5.06	4.88	4.57	3.97	3.69
	6.40	6.50	5.98	6.34	5.40	5.20	5.02	4.22	3.88	3.40
	6.50	6.16	6.00	5.94		5.24	5.00	4.44	4.04	3.63
							4.73	4.61	4.07	3.63
Average.	6.42	6.35	5.91	6.18	5.20	5.17	4.91	4.46	3.99	3.59
$e \times 10^{-8}$.	906	892	835	873	734	730	694	630	563	507

Gold. Cast.

Analysis by Mr. Schluederberg gave 99.0 per cent. gold and traces of platinum. Length = 1.1261 cm. Average width fringes = 110. Density = 19.49. Average coefficient = $1,338 \times 10^{-8}$.

	12.06	12.25	11.15	12.68	11.33	10.21	10.89	10.92	10.19	9.63
	11.83	12.25	11.41	12.18	11.75	10.70	10.10	9.40	9.60	9.73
				12.00	11.09	10.26	10.93	10.83	9.59	9.91
					11.23	11.04	10.96	10.45	10.27	9.73
					11.18	11.19	10.77	9.64		
Average.	11.95	12.25	11.28	12.29	11.31	10.68	10.73	10.25	9.91	9.75
$e \times 10^{-8}$.	1,450	1,486	1,368	1,488	1,373	1,294	1,301	1,242	1,201	1,182

Gold, same as above, but different temperatures. Length = 1.1253 cm. Average width = 100.

The surprising irregularities given by the first test were checked by a second test which gave intermediate points on the curve, thus verifying the first test.

Temp. Interval	294.5° to 284.8°	284.8° to 273°	273° to 263°	263° to 253°	253° to 243°	243° to 233°	233° to 223°	223° to 213°	213° to 201.3°	201.3° to 193°
Plot at	289°	278°	268°	258°	248°	238°	228°	218°	207°	197°
	6.08	6.95	5.80	6.25	5.93	5.31	5.11	6.98	5.95	5.41
	5.90	6.84	5.82	6.16	5.78	5.14	5.48	6.72	6.43	4.99
					6.28	5.01				
Average.	5.99	6.89	5.81	6.21	5.99	5.15	5.29	6.85	6.19	5.20
$e \times 10^{-8}$.	1,500	1,416	1,425	1,524	1,470	1,263	1,298	1,680	1,282	1,520

Iron. Rolled rod.

From Pennsylvania Steel Co. Analyzed by Mr. Blough, formerly of the department of chemistry, and found to contain carbon, 0.058 per cent.; phosphorus, trace; silicon, 0.008 per cent.; manganese, 0.071 per cent. Length = 2.0129 cm. Average width fringes = 200. Density = 7.77. Average coefficient = 925×10^{-8}.

Temp.	283°	263°	243°	223°	203°	183°	163°	143°	123°	103°
	17.34	16.75	16.07	15.84	14.63	13.58	12.40	11.29	9.84	8.16
	17.25	16.73	16.08	15.84	14.61	13.66	12.81	11.13	9.90	8.42
	17.27	16.74	16.07			13.72	12.85	11.43	10.00	8.02
										8.17
Average.	17.29	16.74	16.07	15.84	14.62	13.65	12.69	11.28	9.91	8.19
$e \times 10^{-8}$.	1,172	1,136	1,090	1,074	992	925	860	765	673	556

Platinum. Hammered. Commercially pure.

Specimen loaned by Messrs. J. Bishop & Co., Malvern, Pa. Length = 1.5112 cm. Average width fringes = 500. Density = 22.15. Average coefficient = 815×10^{-8}.

	9.84	9.87	9.82	9.72	9.35	9.07	8.81	8.36	7.88	7.57
	9.93	9.80	9.89	9.69	9.29	9.12	8.88	8.27	8.02	7.58
	9.88	9.81	9.72	9.63	9.33	9.06	8.84	8.26	7.98	7.54
	9.83	9.91	9.89	9.58	9.29	8.94	8.82	8.26	7.97	7.49
Average.	9 87	9.85	9.83	9.65	9.31	9.05	8.84	8.29	7.96	7.54
$e \times 10^{-8}$.	891	889	887	871	841	817	798	748	719	680

Silver. Cast.

Pure, from United States Mint. Length = 1.2805 cm. Average width fringes = 190. Density = 10.43. Average coefficient = $1,740 \times 10^{-8}$.

Temp.	283°	263°	243°	223°	203°	183°	163°	143°	123°	103°
	17.92	17.76	17.58	17.45	16.95	16.47	15.84	15.29	14.37	13.45
	17.94	17.85	17.61	17.49	17.27	16.86	16.00	15.13	14.40	13.53
	17.93	17.78	17.67	17.35	16.77					
Average.	17.93	17.79	17.62	17.43	16.99	16.66	15.92	15.21	14.39	13.49
$e \times 10^{-5}$.	1,910	1,891	1,875	1,855	1,809	1,773	1,696	1,620	1,530	1,435

Soft Solder. Cast.

Commercial "half and half" (tin and lead). Length = 1.8080 cm. Average width fringes = 130. Density = 8.72. Average coefficient = 2301×10^{-8}.

	33.10	33.75	32.75	31.75	31.87	29.75	29.43	28.43	27.71	26.50
	33.80	33.30	32.70	31.85	31.25	30.00	29.80	29.00	27.64	26.60
						29.52		28.10		26.96
								29.00		26.43
								28.55		
								28.12		
Average.	33.45	33.52	32.72	31.80	31.56	29.76	29.61	28.53	27.68	26.62
$e \times 10^{-5}$.	2,515	2,530	2,470	2,409	2,388	2,255	2,245	2,162	2,080	2,020

Tin. Cast.

From Eimer and Amend, marked chemically pure. Length = 1.4332 cm. Average width fringes = 220. Density = 7.32. Average coefficient = $1,585 \times 10^{-8}$.

Since tin is meta-stable at ordinary temperatures and is subject to a slow change to a less dense "gray tin," it was watched carefully, but no great irregularity was observed.

	19.00	18.33	17.92	18.10	16.90	16.50	15.50	15.64	14.60	14.20
	19.29	19.18	18.76	18.23	16.64	16.21	15.00	15.68	14.69	14.37
	19.13	18.90	18.00	18.00	17.12	16.19	15.90	15.49	14.65	14.24
	19.00	18.32	18.27							
Average.	19.10	18.68	18.24	18.11	16.89	16.30	15.47	15.60	14.65	14.27
$e \times 10^{-5}$.	1,832	1,780	1,746	1,729	1,609	1,554	1,473	1,488	1,395	1,361

Quartz Glass.

Cut from tubing. Clear, but not transparent. Length = 1.4917 cm. Average width fringes = 250. Density = 2.01. Average coefficient = -4.25×10^{-8}.

Temperature.	273°	233°	203°	183°	153°	113°
	1.00	.66	.09	−.06	−.70	− 1.49
	1.01	.70	.07	−.10	−.60	− 1.44
	.99	.61	.06	−.06	−.65	− 1.48
	1.01		.02		−.66	− 1.48
	1.01					
Average.	1.005	.66	.06	−.07	−.65	− 1.47
$e \times 10^{-8}$.	46.0	30.2	5.48	−6.40	−29.8	−67.2

Quartz Glass.

Cut from mercury vapor lamp. Clear and transparent as glass. Length = 1.4591 cm. Average width fringes = 300. Density = 2.20. Average coefficient = -8.71×10^{-8}.

Temperature.	273°	233°		193°	153°	113°
	.71	.48		−.10	−.63	− 1.43
	.79	.48		−.11	−.50	− 1.41
	.84	.45		−.09	−.47	− 1.46
	.75	.40		−.11	−.92	− 1.38
Average.	.77	.45		−.10	−.63	− 1.42
$e \times 10^{-8}$.	36.0	21.1		−4.68	−29.5	−66.5

Both quartz specimens changed from a positive to a negative coefficient at about − 80° C. Karl Scheel's equation [1] for quartz glass gives − 46° for the changing point. Both specimens for about 60° on either side of − 80° C. showed this peculiarity: above − 80° when warmed they at first contracted slightly and then expanded, and when cooled they at first expanded slightly and then contracted. Just the converse of this was true for about 60° below − 80° C. This could not be detected at room temperatures nor at liquid air temperatures, nor was it noticed in any other substance. It is as though the coefficient were slightly negative or positive, depending on whether it was warmed or cooled.

Accuracy.

At any temperature worked the resistance of the thermometer can be measured to 0.01 ohm and for a 20° interval this is about ± 0.17 per cent.

[1] Verh. d. Phys. Ges., 9, 1907.

Fringes can be measured to 0.02 fringe and in some cases to 0.01 fringe so that in a 5 fringe interval it would be ± 0.4 per cent. For most specimens the intervals had more than five fringes.

Lengths of specimens can be measured to 2 parts in 10,000 or ± 0.02 per cent.

Combining these by square root of sum of squares gives an error of ± 0.44 per cent. and these errors individually are all compensating errors. About the only error which might be introduced which is not compensating is change in wave-length which would be due to leakage. Considerable work was done on the index of refraction of air, oxygen, nitrogen and carbon dioxide at different temperatures in May, 1906, with this apparatus, which with further work will be published later. It was found that if the pressure actually changed 0.1 cm. during a 20° interval at the lowest temperature the error introduced due to change in wave-length would be only about 0.1 per cent. But no such change of pressure is allowed for a stroke or two of the mercury pump is made before each reading so that the pressure remains the same within one or two hundredths of a cm.

The above sources of error are thought to be the only ones inherent to the method. Individual intervals for some specimens, however, show differences among themselves far in excess of the above indicated errors. Generally it was tried to compensate these discrepancies by several measurements. Some of them may be due to individual peculiarities of the substance. Zinc, for example, shows actual sudden jumps of fringes at some temperatures. Data for it will be given in a future paper.

Considering these discrepancies it is thought that all results are accurate within 2 per cent. and the average coefficients between 293° and 93° within 0.1 per cent.

The work will be continued for other substances and alloys.

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THE EFFECT OF TEMPERATURE UPON THE DISCHARGE OF ELECTRICITY FROM METALS ILLUMINATED BY ULTRA-VIOLET LIGHT.

By George Winchester.

I. Introduction.

THE influence of temperature upon the discharge of electricity from metals illuminated by ultra-violet light has been studied by Hoor,[1] Stroletow,[2] Elster and Geitel,[3] Righi,[4] Zeleny[5] and J. J. Thomson.[6] The early investigations did not produce results capable of definite interpretation because in those experiments a gas surrounded the metal and its influence and effects were unknown. Hoor and Stroletow obtained conflicting results — Hoor found that the current from a zinc surface decreased as the temperature rose and Stroletow found the current rising with the temperature. Elster and Geitel found that the current from zinc was independent of the temperature. Righi found that the positive charge taken up by a hot plate was greater than that taken up by a cold plate; air currents set up by the hot metal may have influenced this result. Zeleny's results from experiments upon iron and platinum at atmospheric pressure bring out several interesting phenomena yet they shed no light upon the question of whether or not there is any change in the photo-electric effect due to a change in the temperature of the molecule, because of the unknown influence which the surrounding gas might have on the results. Elster and Geitel found that the current from a potassium surface in a good vacuum when its temperature changed from 20.3 to 50.3 was increased

[1] Hoor, Wien. Berichte, Vol. 97, p. 719, 1888.
[2] Stroletow, Comptes Rendus, Vol. 108, p. 1241, 1889.
[3] J. Elster and H. Geitel, Wied. Ann., Vol. 48, p. 625, 1893.
[4] Righi, Atti. Ist. Ven., 7-Mem. 11.
[5] Zeleny, Phys. Rev., Vol. 12, p. 321, 1901.
[6] J. J. Thomson, Conduction of Electricity through Gases, p. 239.

nearly 60 per cent. J. J. Thomson in his book on Conductivity of Electricity through Gases refers to some work by himself in which the alkali metals were heated to a temperature of about 200° C. in a good vacuum ; he also found a very great increase in the current. E. Ladenburg[1] investigated the relative rates of discharge from the ordinary metals in a high vacuum but did not alter the temperature. He found that the rate of discharge depended much upon the condition of the electrode ; *i. e.*, upon the method and degree of polish of the surface ; also, that the value changed with the time. Just what change is produced in the electrode so as to alter the photo-electric effect he does not make clear.

The principal object in the present investigation has been to find out whether or not there is any temperature effect when the disturbing influence of the surrounding gas has been removed ; *i. e.*, with an increase of energy of translation of the molecule is there also an increase of subatomic energy shown by an increased photo-electric effect ? In the preparation of the metal disks as well as throughout the experiment, care was taken that all the different metals received the same treatment, and that the ultra-violet light acted upon the different metals under exactly similar conditions.

II. Description of Apparatus.

The apparatus which is shown in Fig. 1 consisted of a large glass bulb (Fig. 2) within which was mounted a circular aluminium disk used as a support for the different metals employed. The wheel was under magnetic control so that any metal might be rotated to a position in front of the source producing the ultra-violet light. The wheel was mounted on agate bearings and was balanced so that it would remain at rest when rotated into any position. Across the back of the wheel was fastened a small bar magnet to facilitate its rotation, which was accomplished by means of a large horse-shoe magnet ; in order that the wheel might be connected to the electrode outside the bulb, a platinum spring was used to place it in contact with the frame as is shown in the figure. The frame was in direct connection with the electrode. Metal disks of copper, nickel, iron, zinc, silver, magnesium, lead, antimony, gold, aluminium and brass,

[1] E. Ladenburg, Ann. d. Phys., Vol. 12, p. 558, 1903.

each disk one centimeter in diameter, were attached to the circumference of the wheel. The disks were screwed to the aluminium wheel and polished with dry emery so as to be free from the presence

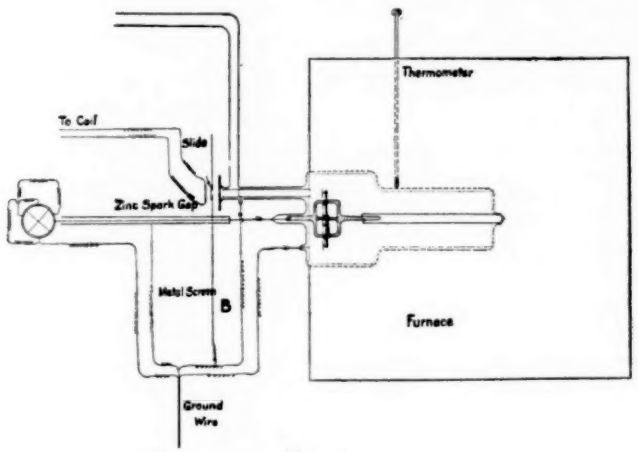

Fig. 1.

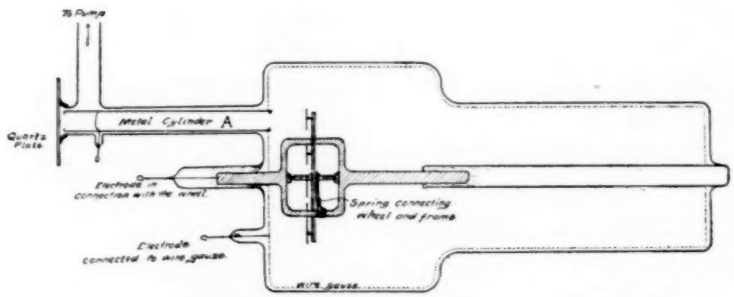

Fig. 2.

of any oils which might influence the photo-electric effect. They were finally heated, washed in alcohol, heated again, and mounted within the glass bulb without having their surfaces touched. Around the inner surface of the bulb and enclosing the wheel was placed a brass wire gauze which could be connected to earth or to the electrometer whichever was desired. In order that the different disks might occupy the same position with respect to the source of light and the wire gauze when they were exposed to the influence of the light, they were made to rotate in the same plane to within one one-thousandth of an inch or less. The ultra-violet light was admitted

to the bulb through a quartz plate sealed over the end of a projecting tube about 10 cm. long; within this glass tube was placed a co-axial brass cylinder, *A*, whose inside surface was coated with a dull black finish to avoid reflection and to prevent any photo-electric effects from its surface; its ends were closed with stops having openings of about 8 mm. diameter; it could be connected to earth or to the electrometer just as in the case of the wire gauze; the cylinder was placed close to the wheel so that the light would strike the disk which was being used and no part of the aluminium wheel would receive any light. Between the quartz plate and the source of light was a large metal screen, *B*, connected to earth, with an opening for the light; the opening was furnished with a shutter with which it was closed before an observation was taken and while the tests for inductive effects, etc., were being made.

The pump used to exhaust the bulb was a type of mercury pump with all connections of glass and capable of producing a good vacuum. The measurements on the photo-electric effect were made at pressures of .00001 mm. or less, as recorded by a McLeod gauge; the photo-electric effect was not influenced by pressures of this value as could be determined by working the pump until no gas could be pumped out and comparing values. Whenever a new bulb was sealed on, the temperature was always raised to about 400°C. at intervals; and this was continued until no more gas was given off; that this condition was realized was indicated by the fact that the gas from a bulb of 1,500 c.c. would collapse in a capillary tube of 1 mm. bore under a pressure of less than a centimeter of mercury; at the same time the reading of the gauge would show pressures ranging from .00001 mm. to zero pressure. This is merely intended to show that the pressure was below that which would influence the photo-electric effect. The pump was always kept running throughout a series of observations to be sure that no gas collected in the bulb.

The electric furnace which was used to regulate the temperature was a box made of sheet-metal with heavy asbestos linings. Porcelain spools were covered with asbestos, wound with iron wire, and supported on the four sides and the bottom of the box. When the furnace was joined to a 220-v. circuit a temperature of 500°C. could

quickly be obtained. The amount of current was regulated by rheostats in series with the furnace.

A spark between zinc electrodes was used as a source of ultra-violet light. At first considerable difficulty was experienced in obtaining results which would check within ten or twenty per cent. under apparently the same conditions. It was thought that the variations in the deflections of the electrometer needle could be accounted for by the unsteadiness of the hammer of the induction coil. Several devices, including a mercury-turbine interrupter, were substituted for the hammer but without success. Finally it was noted that the irregularities could be accounted for in the leak of the electrometer itself. When this was corrected the readings could be duplicated to within one or two per cent. A variable capacity was used in connection with the coil so that the character of the spark might be somewhat altered when necessary.

As a source of constant potential a battery of 900 elements was used; this was made of copper-zinc strips in water with one or two drops of H_2SO_4 added to every liter of water. This source of potential would remain constant for months at a time — or so long as the battery was kept dry and well insulated. This battery was used to charge the needle of the electrometer and the metal disks.

One of the most serious difficulties encountered in carrying out this experiment was due to the humidity of the atmosphere during the summer months, when the work was done. This was so great that the use of any of the best insulators was of no avail in retaining the small charges generated. Sulphur would insulate extremely well, but its use had to be dispensed with because it could not be made to form a plug that would not leak air when a high vacuum was obtained. The small amounts of electricity generated would leak over a length of 20 cm. of small quartz rod, even when heated and a small stream of dry air kept blowing over the rod. As a last resort it was decided to enclose the entire apparatus in a room made of building paper and attempt to dry the enclosure. The room was built and trays of H_2SO_4 and $CaCl_2$ were distributed throughout. Also a stream of dry air was kept continually flowing into the room. Tubes were inserted in the walls for the observer to breathe through. That this method of stopping the leak was capable of giving quite satisfactory results will be seen farther on.

III. OBSERVATIONS OF THE RELATION BETWEEN THE RATE OF DISCHARGE AND THE TEMPERATURE OF THE METAL.

In making these observations the wheel was joined to one pair of quadrants of the electrometer, while the other pair was joined to earth. The quadrants and the wheel were connected to the negative pole of a battery of 20 cells (20 volts), the positive pole being earthed, and a certain deflection was produced; the connection was then broken, the induction coil started, and finally the shutter was raised and the light was left on for 10 seconds; the needle came to rest again in perhaps 60 or 90 seconds, and the difference between the two deflections was taken as the leak due to the photo-electric effect. The leak of the electrometer was so small as to be inappreciable in comparison with that due to the light, so long as the temperature remained below 100° C. At temperatures above 100° C. a correction had to be made to eliminate the natural leak, which at 170° C. had become so large as to mask entirely the effect of the light. The cause of this leak is unknown. Positive and negative charges would disappear with equal readiness.

When an observation was made a certain temperature was established by means of the furnace and held steady by altering the current; then the wheel was rotated and the different metals were exposed in turn to the influence of the ultra-violet before the quartz window. Sometimes two or more series of readings would be taken before the temperature was changed in order to see if the values could be duplicated. The observations could be checked over a time of two weeks as closely as they could at any one time. The following table shows a representative series of observations. Here the metals are arranged in the order of their ability to discharge negative electricity; about eight times out of ten the order was as given in the table; at other times the nickel and the brass changed places, as may be seen by looking at the table. Their values are so nearly alike that conditions could not be sufficiently controlled to discriminate between them. It would perhaps seem that if any of the metals were to change places in the table, due to their nearly equal values, it would be the brass and the copper, instead of the brass and the nickel. The order of the other metals was always the same. Also, the results were just the same, whether

the wheel was joined to the electrometer, and the gauze and metal cylinder earthed, or the wheel earthed and the gauze and cylinder joined to the electrometer.

TABLE I.

Metal.	Rate of Leak in Scale Divisions.							
	$T=25°$	$T=35°$	$T=40°$	$T=50°$	$T=65°$	$T=80°$	$T=100°$	$T=125°$
Cu	25.10	25.15	25.20	25.25	25.00	25.05	24.80	24.75
Au	24.70	24.60	24.55	24.70	24.80	24.75	24.40	24.00
Ni	24.00	23.96	23.98	23.90	24.05	23.90	23.55	23.40
Brass	23.80	23.85	23.95	24.00	23.85	23.90	23.40	23.40
Ag	17.16	17.20	17.15	17.20	17.10	17.00	16.90	16.77
Fe	16.40	16.25	16.30	16.20	16.36	16.55	16.15	16.00
Al	14.90	15.00	14.85	14.86	15.06	14.90	14.40	14.55
Mg	11.00	11.12	11.10	11.05	11.00	10.97	10.90	10.90
Sb	4.00	4.00	4.10	4.00	4.00	4.00	3.90	3.95
Zn	1.20	1.30	1.31	1.35	1.20	1.20	1.24	1.10
Pb	.90	.90	.90	.88	.90	.90	.90	.90

The table shows that up to 100° C. if there is an increase in the rate of discharge of electricity from these metals due to the increased temperature, it lies within the errors of the experiment. The readings at 100° C. and 125° C. were corrected for the temperature leak as explained below and are therefore weighted with a larger error than are the readings at the other temperatures.

In some experiments made in May, 1905, by Professor R. A. Millikan and the writer concerning the effect of ultra-violet light with change of temperature upon an aluminium surface, the temperature could be raised to at least 350° before any very decided effect due to the temperature was noticed, yet some other disturbing element was present which hindered very consistent results being obtained. At 400° the metal would not hold a charge at all. Below in Table II. is given a series taken Tuesday, May 30, 1905. It will be seen from the table that the results check over the entire range of temperature with about the same degree of accuracy that several readings do at any one temperature. It was concluded from observations like these that if there was any influence of the temperature on the photo-electric effect it must be rather small. The method employed in taking these observations was to observe the leak of the electrometer for 30 seconds, then

throw the light on for 15 seconds and take the deflection at the end of 45 seconds and finally to observe the leak during another 30-second period. The sum of the two electrometer leaks during the 30-second periods was then subtracted from the change in the electrometer readings during the intervening 60-second period and this remainder taken as the leak due to the light.

TABLE II.

Temp.	Deflection in Scale Divisions.	Temp.	Deflection in Scale Divisions.	Temp.	Deflection in Scale Divisions.	Temp.	Deflection in Scale Division.
49°C.	18.0	272°C.	14.3	304°C.	15.5	78°C.	20.3
50	16.3	275	14.2	310	15.8	75	19.3
50	17.4	280	14.0	343	15.4	73	20.6
100	16.0	286	13.8	343	17.0	69	20.9
142	14.2	290	15.1	256	16.9	67	20.8
169	17.0	292	15.4	200	18.0	64	21.6
191	16.5	295	15.2	170	17.1	61	20.4
205	16.6	296	16.2	143	17.3	58	20.5
215	16.3	297	15.7	130	18.1	54	20.9
220	17.2	298	15.9	120	17.4	54	20.1
233	12.9	300	15.7	112	19.2	53	20.4
246	14.2	301	16.3	105	17.5	52	20.8
254	13.6	300	15.8	100	17.6	52	19.6
260	14.0	301	16.6	95	19.2	53	17.3
264	13.9	301	15.3	89	18.7	53	18.8
269	13.3	303	15.7	85	21.2	53	17.0

The variations in the readings shown in this table are doubtless wholly due to variations in the electrometer leak; at least when this source of error was eliminated in the manner described the values checked as shown in Table I. It would thus appear that there is no temperature effect on aluminium when the temperature is raised as high as 340° C. The bulb containing this disk went through a long series of heatings as high as 500° C. while the pump was kept running; an induction coil was run intermittently on the tube at the same time; this was carried on for at least ten days or two weeks before any observations were made. This possibly accounts for the fact that so high a temperature was reached before any discharge due to the heat was present; the large bulb with which the present observations were made could not be con-

veniently sparked with a coil; neither could it be heated as high as 400° C. except for periods of short duration.

On a separate bulb containing a small aluminium disk very consistent results were able to be secured to as high a temperature as 195° C. showing no increased discharge due to the temperature of the metal. This tube could be heated much hotter and the temperature was held at about 500° C. until no more gas appeared at the pump; in general, it was noticed that the more the tube was heated while the pump was kept running and the more the coil was used on the tube, the more consistent were the results and the higher the temperature that could be reached before the heat discharge influenced the results.

The relative values of the discharging powers of the different metals follows none of the ordinary series of metals; in general, it somewhat closely coincides with the melting point series, and yet it is hardly probable that the two phenomena have any necessary connection.

The following table showing the melting points of the metals and their order in the photo-electric series as determined in this investigation is inserted for comparison. The melting points are taken from J. Castell-Evans' physico-chemical tables and are those given by the author as the most probable values.

TABLE III.

Metal.	Melting Point.	Discharge Rate.
Cu	1050° C.	25.10
Au	1050	24.70
Ni	1450	24.00
Brass	——	23.80
Ag	968	17.16
Fe	——	16.40
Al	625	14.90
Mg	750	11.00
Sb	632	4.00
Zn	433	1.20
Pb	326	.90

Elster and Geitel arrange the metals of lower melting points in a similar series with reference to their power of discharging negative electricity in which just the opposite is shown, namely, the

lower the melting point the higher the value for the discharging effect. The following is their series to which has been attached the melting points.

TABLE IV.

Metal.	Melting Point.	Metal.	Melting Point.
Ru	38.5° C.	Li	180° C.
K	62	Mg	750
K + Na	—	Th	294
Na	97	Zn	433

IV. INFLUENCE OF TEMPERATURE UPON THE POSITIVE POTENTIALS ASSUMED BY THE DIFFERENT METALS.

In obtaining the positive potentials assumed by the different metals when exposed to ultra-violet light, the wire gauze and the metal cylinder were connected to earth; the wheel and its attached pair of quadrants were kept at zero potential, and the shutter in the screen, before the quartz window, was kept closed until an observation was ready to be made. Then the earth was removed from the electrometer and later the shutter was raised and the light kept on until the needle assumed a steady position. The time required for the needle to come to a permanent deflection varied with the different metals. If the source of light was placed farther away the time was longer but the final position of the needle was the same. Also, if the capacity was changed so as to alter the nature of the spark there was only a variation in the time necessary for the metal to assume this same potential. The average time necessary for the metals to assume the potentials given below was about 90 seconds; while at the same time one was unable to detect any leak in the electrometer over a period of five or six minutes. The values of the deflections could be repeated again and again to within less than one per cent. with those metals which assumed the higher potentials, the error being greater for the metals of lower potentials. That these readings were able to be duplicated to within so much smaller per cent. of error than in the previous set of observations on the rate of leak (Table I.) is probably due to the fact that in the observations on rate of leak the light was acting upon the metal for a comparatively short time, and might be less effective during one

ten-second interval than during another equal interval; while in this potential series, if the light was less effective during the first part of the period it would assume its normal value before the close of its period.

For the lower values of the temperature the readings could be made very accurately, but as soon as the discharge of electricity due to the heated condition of the metal was at all apparent, the potentials were reduced. The values of the potentials were checked up to about 100°C. and no increase was noticeable. The agreement between the different values of the potential (shown in Table V.) at different temperatures was of a somewhat higher order of accuracy than that shown in Table I. on the values of the discharge effect.

TABLE V.

Metal.	Deflection.					
	26°	40°	55°	60°	80°	95°
Ag	47.6	47.6	47.7	47.5	47.5	47.3
Fe	43.5	43.4	43.4	43.7	43.2	43.2
Au	43.1	43.2	43.1	43.0	43.1	43.0
Brass.	41.7	41.6	41.8	41.7	41.8	41.9
Cu	40.3	41.2	41.1	41.0	41.0	40.9
Ni	40.0	40.0	40.1	40.1	39.8	40.1
Mg	29.8	29.9	29.9	30.0	29.7	29.9
Al	26.2	26.2	26.2	26.1	26.3	26.2
Sb	14.0	14.1	14.0	14.1	14.1	14.0
Zn	7.0	7.0	7.1	6.9	6.8	6.7
Pb	00.0	00.0	00.0	0.00	00.0	00.0

Zeleny,[1] in his investigation on the effect of temperature upon platinum and iron exposed to ultra-violet light and surrounded by air at atmospheric pressure, observed a peculiar hysteresis effect, the discharges being greater when the metals were cooling than when they were rising in temperature. That particular effect was not noticed in these observations but a somewhat similar effect was observed; namely, that observations could be made at 50° to 60° higher when the temperature was rising than when it was falling. That is, if in the potential series 95° C. was the highest temperature at which good results could be obtained when the temperature was

[1] Zeleny, PHYSICAL REVIEW, Vol. 12, p. 321, 1901.

rising and if then the temperature was increased to say, 200° C., and the apparatus left to cool, then the first value that the heat discharge did not mask would be at about 40° or 30° C.

I desire to take this opportunity of acknowledging my obligations to Professors Michelson and Millikan for continued suggestions and assistance throughout this investigation, a considerable portion of which has been conducted in collaboration with the latter.

Ryerson Physical Laboratory,
University of Chicago,
September 1, 1906.

TEMPERATURE CONTROL OF THE CHELTENHAM MAGNETIC OBSERVATORY U. S. COAST AND GEODETIC SURVEY.[1]

By J. E. Burbank.[2]

A FULL description of the Cheltenham Magnetic Observatory is contained in the Annual Report of the Superintendent of the Coast and Geodetic survey for the year 1902, Appendix No. 5.[2]

The survey had maintained a magnetic observatory at Los Angeles, California, from 1882 to 1889 and also at San Antonio, Texas, during the period 1892 to 1895. In addition to the experience already gained a careful study was made of the best known foreign observatories and the methods employed to control temperature changes in their variation buildings.

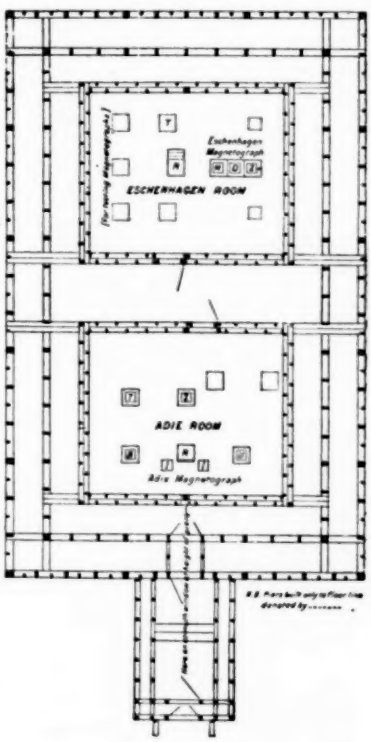

Fig. 1.

As a result of this study it was decided to build the Cheltenham Observatory entirely above ground and provide sufficient heat insulation to reduce the annual range of temperature inside to a reasonably small amount, such a building would furthermore be free from the excessively humid conditions present in an underground structure.

The observatory is essentially a building within a building, the

[1] Communicated with permission of the Superintendent Coast and Geodetic Survey.

[2] Observer in Charge of Cheltenham Magnetic Observatory.

[3] See also Journal Terrestrial Magnetism, March, 1903, pp. 11–29.

inner portion being divided into two separate closed rooms known respectively, as the Adie and Eschenhagen rooms from the types of magnetographs which they contain. The walls of the outer building and also of the inner rooms are packed with dry pine sawdust.

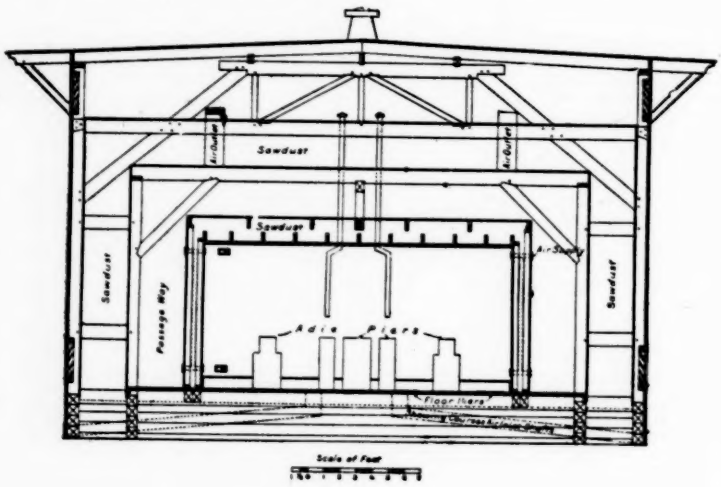

Fig. 2.

The wall thickness was designed to reduce the mean annual range of temperature, about 25° C. outside, to 4 or 5° C. inside, assuming no artificial heat and no openings in the walls for entrance or ventilation. For a plan of the variation building see Figs. 1 and 2.[2]

The inner rooms are each 19½ by 16 by 9 feet inside measure and are entered through refrigerator pattern doors from a central corridor, 5½ feet wide, extending between them, the sides and top are surrounded by an air space about 3 feet wide. In each room there are four small air shafts 3 inches square communicating with the air space around the rooms, these are fitted with slides and are generally kept closed. In the Adie room are three 3-inch ventilating tubes directly over the lamps and extending up into the attic, to carry off the hot air and products of combustion; during the winter these lamp ventilators are disconnected directly over the top of the room. A similar ventilator is provided in the Eschenhagen room. The doors, air shafts and lamp ventilators are the only openings in the inner rooms.

In the walls of the outer building there are four inlet shafts 5 by 10 inches and 16 feet long so placed that the air enters from the outside of the building near the ground and discharges through floor gratings into the passage-way at either end of the building. Four corresponding air outlet shafts 6 by 10 inches extend through the upper layer of sawdust into the attic. These air shafts are all provided with close fitting shutters and are opened only during the warm summer months. The outside wall has an 8-inch air space between the sawdust packing and the weatherboarding, connecting with the attic space, and provided with six ventilating windows, three on each side of the building near the ground; during the hot weather these air shafts admit a cooling draft around the building during the night hours.

Entrance to the building is from the south side through a vestibule 10 feet by 13 feet 8 inches with walls containing two feet of sawdust packing. This vestibule is divided into three compartments, or air-locks, by four sets of close fitting doors.

The insulation of the building, beginning on the outside is: pine weatherboarding, 8-ply building paper, 1-inch pine sheathing, 8-inch air shaft, 1-inch pine sheathing, 8-ply paper, 3 feet pine sawdust, 8-ply paper, seven eighths inch pine ceiling, 3 feet 2 inches air space of passage-way, seven eighths inch pine ceiling, 8-ply paper, 1 foot pine sawdust, 8-ply paper, seven eighths inch pine ceiling. Beginning at the roof and going down the insulation is: gravel and pitch roof, three thickness tar paper, 1-inch pine sheathing, 3 feet 8 inches air space, 1-inch rough pine floor, 3 feet pine sawdust, 8-ply paper, seven eighths inch pine ceiling, 3 feet air space above rooms, 1-inch rough pine floor, 1 foot 6 inches sawdust, 8-ply paper, seven eighths inch pine ceiling, seven eighths inch pine floor, 8-ply paper, 1-inch pine under floor, 3 feet pine sawdust, 6-inch to 8-inch layer of gravel, 2 feet 8 inches of earth.

During the summer months, from about June 1 until October 15, the attic shutters are kept open so that a comparatively free circulation of air is obtained under the roof. During the night hours and on cool cloudy days the inlet shafts and the lower side shutters are opened allowing circulation of the air in the passage-way around the rooms and in the air shafts in the outside walls.

The Adie magnetograph consists of three photographic recording variometers each of which is lighted by a central draft student lamp. These lamps run continuously throughout the year, being kept as low as possible during the summer, they consume approximately 1 ½ pints of common kerosene oil per 24 hours.

In the Eschenhagen room only one small lamp is used to light all three variometers. It uses approximately ½ pint of oil in 24 hours. The presence of these lamps necessitates a temperature, in summer, considerably higher than the outside mean temperature, and in order to diminish the annual range it is necessary to maintain a correspondingly high temperature in winter. On account of the irregular nature of this source of heat any accurate control is out of the question, and it is not probable that any automatic device could be made to work satisfactorily.

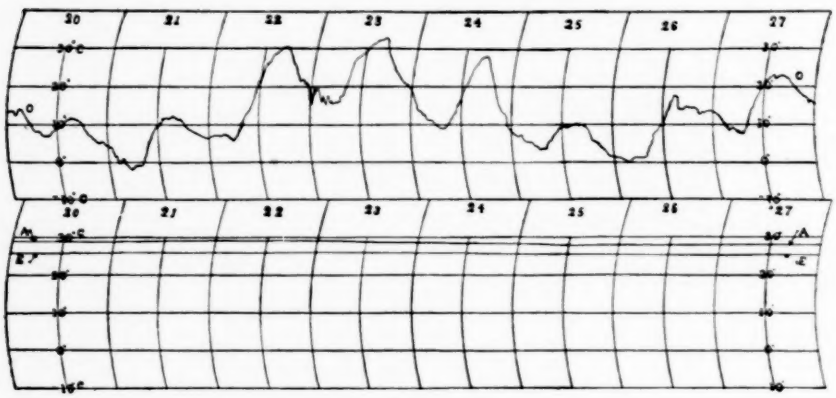

Fig. 3.

Thermograms Cheltenham Magnetic Observatory, March 20–27, 1907, *O*, Outside, *A*, Adie Room, *E*, Eschenhagen Room.

During the winter months additional lamps similar to the Adie lamps are used to heat the passage-way around the Eschenhagen room, in the coldest weather four or five are used continuously.

As a result of the past four years' record it has been shown that there is no diurnal variation, the occasional changes of one or more tenths of a degree being due to irregularities in the control of the heating lamps or ventilators. These irregularities are most common in the Adie room in the winter when the lamp ventilators are shut

off, at such times the change may amount to three or four tenths of a degree in a single night, due largely to irregularities in the Adie lamps. The changes in the Eschenhagen room rarely exceed one tenth of a degree in 24 hours.

It has been found that a temperature of about 28° C. in the Adie, and 25° C. in the Eschenhagen room, is best suited to present conditions.

It has been found repeatedly that a sudden change, amounting to 20° C. or 30° C. outside, can be entirely counteracted by the judicious use of heating lamps in winter and ventilators in summer, and in the case of long continued cold weather the effects are felt only gradually inside and need never amount to more than a few tenths of a degree.

As an example of the effect of a sudden change of temperature outside, see Fig. 3, which is copied from the thermograms obtained in the Adie room (*A*), in the Eschenhagen room (*E*), and outside (*O*), during the period March 20 to March 27, 1907. During this time there was an exceptionally large change of temperature, from — 2° C. at 5 a. m. March 21, to 33° C. at 4 p. m. March 23, a change of 35° C. in 59 hours. The thermograph pen in the Eschenhagen room, which is normally four tenths of a degree higher than the tnstrument thermometers during this period, due to its setting when ihe sheet was changed, rose from 25.6° on March 20, to 25.9° on March 23, and fell to 25.4° on March 27. The rise was due largely to extra lamps used during ventilation of the passage-way on March 21. During the period March 24 to 27, when the effect of the hot wave should have been felt, the temperature inside the building was falling on account of the control of the heating lamps. In the Adie room the effect was similar but somewhat larger, there the decrease began on the 22d. In this particular instance the effect of the hot wave was overestimated and insufficient allowance was made for the low temperature of the sawdust packing.

In addition to a Richard thermograph in each room, each of the magnetic variometers is supplied with a thermometer and all of these thermometers are read daily at 8 a. m. and 4 p. m. From these readings, by aid of the thermograms, a table of hourly values is constructed, and from these hourly values the daily and monthly

mean values are derived. Since the declination results require no temperature corrections the mean of the temperatures of the two ntensity variometers is taken as the magnetograph temperature.

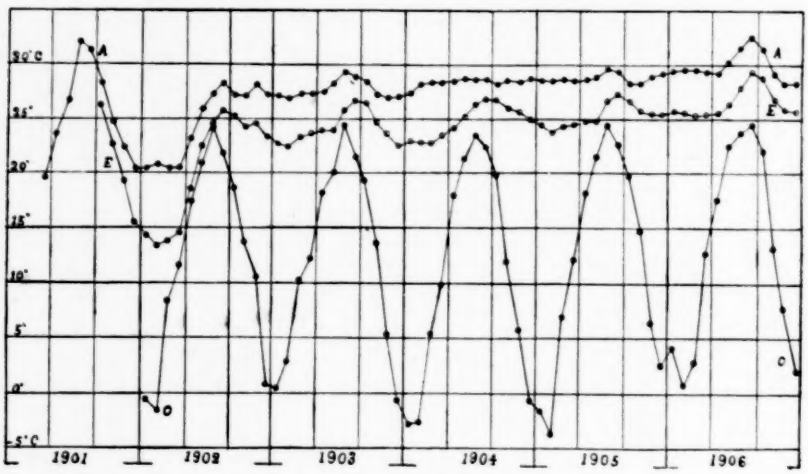

Fig. 4.

Monthly Mean Temperatures Cheltenham, Magnetic Observatory, 1901 to 1906. Adie Room (*A*), Eschenhagen Room (*E*), Outside (*O*).

In Fig. 4 are plotted the monthly mean temperatures, in degrees centigrade; of the Adie magnetograph (curve *A*), of the Eschenhagen magnetograph (curve *E*), and outside (curve *O*), from the beginning of each series of observations to the end of 1906. For the present comparison the monthly mean outside temperatures are derived from the daily maxima and minima. The mean derived in this way will doubtless differ somewhat from the monthly mean value of the hourly readings of the thermograms.

As will be noted the curves for the years 1901 and 1902 show that the method of temperature control was in the experimental stage and a comparatively large annual range was obtained. Beginning with 1903 a more careful control was exercised, especially over the Adie room which shows a much smaller annual range than the Eschenhagen. The curves also show a gradual upward tendency during the years 1905 and 1906, corresponding to the upward tendency of the outside temperature during those years. The lag of the inside temperature change over that outside is also shown in the case of the summer maxima in the Eschenhagen room.

The temperature ranges in degrees centigrade for the various years are given in the following table:

Annual Temperature Range in Degrees Centigrade.

	1902.	1903.	1904.	1905.	1906.
Adie room.	7.79	2.39	1.30	1.43	4.46
Eschenhagen room.	12.37	4.34	4.12	3.38	4.05
Outside (approximate).	26.00	25.00	26.00	28.00	23.70

Very little is yet known regarding the effect of changes of humidity on the quartz fibers supporting the Eschenhagen magnets, the effect of such changes on the silk fibers supporting the Adie magnets is undesirable. Beginning with 1902, observations of relative humidity have been made daily by means of a sling psychrometer, outside at 7:50 a. m., in the variation rooms between 8:10 and 8:20 a. m. The results show a slow but regular change inside the ob-

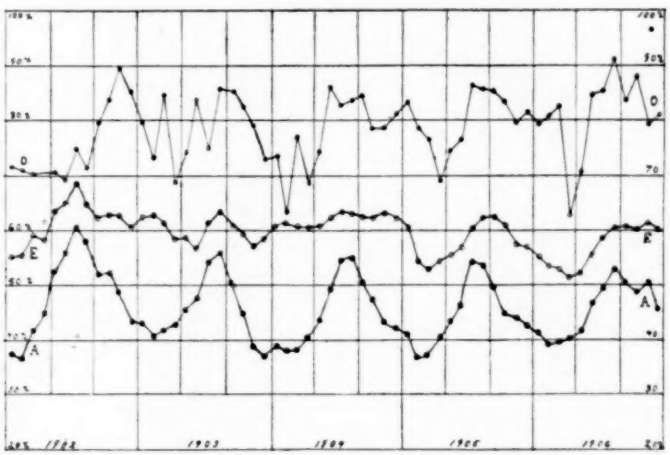

Fig. 5.

Monthly Mean Values of Relative Humidity, Cheltenham Magnetic Observatory, 1902 to 1906. Outside (*O*), Adie (*A*), Eschenhagen (*E*).

servatory with a maximum in summer and a minimum in winter, when the building is kept closed. The monthly range is seldom more than 2 or 3 per cent., while the annual range in the Adie room is about 15 to 18 per cent.

In Fig. 5 are plotted the monthly mean values of the relative

humidity, in per cent. for the Adie room (*A*), for the Eschenhagen room (*E*), and outside (*O*). The monthly mean of the relative humidity outside has no very great significance beyond showing the greater humidity in summer. It will be noted that the relative humidity in the Adie room is much lower than in the Eschenhagen due probably to the presence of the three lamps and the consequently dryer atmosphere. The higher humidity in summer is caused by the method of ventilating the building during cloudy days and during the night hours.

The total yearly consumption of oil at the observatory is approximately 300 gallons of which about 50 gallons is used in the absolute building, leaving about 250 gallons as the total amount used in heating and lighting the variation building. Of this 250 gallons, about 175 are used to illuminate the magnetographs, the remainder being used in the heating lamps in winter. This involves a total cost, for heat and light, of about $25.00 per year.

THE EFFECT OF FREQUENCY UPON THE CAPACITY OF ABSOLUTE CONDENSERS.

By J. G. Coffin.

IT is proposed to construct an absolute air condenser of two circular plates of optically plane glass. Such plates cannot be easily made much larger than 10 cm. in diameter and hence must be placed very near to each other in order to obtain a sufficiently large capacity.

In order to render the two opposing faces of the plates conducting they are to be half-silvered so that light may be seen by transmission through them. They now form practically a Perot and Fabry interferometer [1] and the beautifully sharp fringes seen by transmitted light enable their parallelism and distance apart to be observed with the utmost accuracy at any instant. Knowing the geometrical dimensions of the system, its absolute electrostatic capacity can be calculated by means of a formula due to Kirchhoff,[2] in connection with certain edge-corrections worked out by the writer.[3]

The half-silvered conducting surfaces of the condenser, however, have a very high resistance, and a doubt arises as to whether this condenser is completely charged and discharged when a rapidly varying electromotive force is impressed upon it. The following calculations were undertaken in order to determine the amount of this effect.

Rectangular Plate Condenser.

As a first approximation consider the flow of electricity in a uniform strip of resistance R ohms per cm., and of capacity C farads per cm. (Fig. 1).

[1] Théorie et Applications d'une nouvelle mèthode de spectroscopie interferentielle, Ann. de Phys. et de Ch., Vol. 16, p. 115, 1899. Théorie des Franges des Lames Minces Argenteés, Vol. 12, pp. 459–501, 1897.

[2] Kirchhoff, Gesam. Abhand.

[3] Edge Corrections in the Calculation of the Absolute Capacity of Condensers. P. A. A. S., Vol. 39, No. 19, Apr., 1904.

This represents a condenser made up of two insulated rectangular plates placed parallel to each other. The current enters at the edge AA'. The second plate is connected to earth.

Suppose an impressed alternating electromotive force,

$$V_0 e^{i\omega t}$$

where V_0 is the maximum E.M.F. and ω is the number of alternations in 2π secs., be impressed at one end of the strip.

Fig. 1.

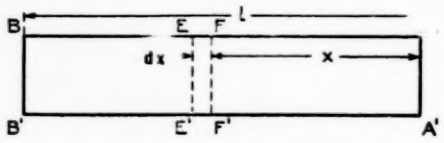

Fig. 2.

Consider an element of the strip of length dx at a distance x from the end AA' (Fig. 2). Let the current, which enters the element at FF' be denoted by I, then the current which leaves at EE' is $I + \partial I/\partial x dx$, and hence the rate of accumulation of electricity in the element dx is

$$I - \left(I + \frac{\partial I}{\partial x} dx\right) = -\frac{\partial I}{\partial x} dx = \frac{\partial}{\partial t}(dQ) = C\frac{\partial V}{\partial t} dx$$

because $dQ = CdxV$, where V is the potential at the element, so that

$$-\frac{\partial I}{\partial x} = C\frac{\partial V}{\partial t}. \qquad (1)$$

Similarly the difference in potential between the two edges of the element EF is

$$V - \left(V + \frac{\partial V}{\partial x} dx\right) = -\frac{\partial V}{\partial x} dx$$

which gives rise to a current of amount $IR\,dx$, so that:

$$IR = -\frac{\partial V}{\partial x}. \qquad (2)$$

We are here neglecting the E.M.F. due to self-inductance which in the cases here considered is *very* small in comparison with the large ohmic E.M.F. IR here assumed.

Eliminating I by differentiating equation (2) with respect to x, there results

$$\frac{\partial^2 V}{\partial x^2} = CR\frac{\partial V}{\partial t}. \tag{3}$$

Let the potential at any point of the strip be given by

$$V = V_0 f(x) e^{i\omega t} \tag{4}$$

where $f(x)$ is to be determined subject to the conditions that when

$$\begin{aligned} &x = 0 \quad f(x) = 1 \\ &x = l \quad f'(x) = 0 \ \text{(equivalent to } \frac{\partial V}{\partial x} = 0\text{)}. \end{aligned} \tag{5}$$

Substituting equation (4) in (3), there results a differential equation for $f(x)$

$$\frac{d^2 f(x)}{dx^2} = i\omega CR f(x) \tag{6}$$

which becomes if we write $-k^2 = i\omega CR$

$$\frac{d^2 f}{dx^2} + k^2 f = 0. \tag{7}$$

The complete solution of this equation is

$$f(x) = Ae^{ikx} + Be^{-ikx} \tag{8}$$

where A and B are the arbitrary constants.

Employing the boundary conditions (5), there results, for the determination of A and B, the equations

$$A + B = 1$$

$$Ae^{ikl} - Be^{-ikl} = 0$$

from which

$$A = \frac{e^{-ikl}}{e^{ikl} + e^{-ikl}} \qquad B = \frac{e^{ikl}}{e^{ikl} + e^{-ikl}}$$

hence $f(x)$ is completely determined and

$$V = V_0 \left(\frac{e^{ik(l-x)} + e^{-ik(l-x)}}{e^{ikl} + e^{-ikl}} \right) e^{i\omega t}. \tag{9}$$

The total quantity of electricity in the condenser at any time is found by integrating $VCdx$ over the whole strip, thus :

$$Q(t) = \int_0^l CV dx = CV_0 e^{i\omega t} \int_0^l f(x) dx$$

$$= \frac{CV_0 e^{i\omega t}}{ik} \frac{e^{ikl} - e^{-ikl}}{e^{ikl} + e^{-ikl}} = Q_0 e^{i\omega t} \text{ (say).} \qquad (10)$$

We are principally interested in the maximum value of $Q(t), Q_0$ which is the coefficient of $e^{i\omega t}$ in the above equation. If the plates were good conductors this value of Q_0 would be CV, hence the ratio of the apparent capacity (maximum) to its actual capacity for a steady charge is

$$\gamma = \frac{1}{ikl} \frac{e^{ikl} - e^{-ikl}}{e^{ikl} + e^{-ikl}} = \left[\frac{1}{kl} \frac{\sin (kl)}{\cos (kl)} \right], \qquad (11)$$

γ is thus the number which tells us how much of the ordinarily assumed capacity is approached for any value of $k^2 = -i\omega CR$.

To realize the expression for γ which in k contains the $\sqrt{i}$ put

$$h(1 + i) = ki.$$

In general the square of the absolute value of any imaginary expression (γ), is the product of it by its conjugate $(\bar{\gamma})$; the conjugate being obtained by replacing i wherever it occurs by $-i$, in the original expression. Hence

$$\gamma\bar{\gamma} = |\gamma|^2 = \frac{1}{l(h + ih)} \frac{e^{hl}e^{ihl} - e^{-hl}e^{-ihl}}{e^{hl}e^{ihl} + e^{-hl}e^{-ihl}} \times \frac{1}{l(h - ih)} \frac{e^{hl}e^{-ihl} - e^{-hl}e^{ihl}}{e^{hl}e^{-ihl} + e^{-hl}e^{ihl}}$$

$$= \frac{1}{2h^2l^2} \frac{(e^{2hl} + e^{-2hl}) - (e^{i2hl} + e^{-i2hl})}{(e^{2hl} + e^{-2hl}) + (e^{i2hl} + e^{-i.hl})} \qquad (12)$$

$$= \frac{2}{(2hl)^2} \frac{\cosh 2hl - \cos 2hl}{\cosh 2hl + \cos 2hl}$$

Putting a single symbol λ for $2hl$, there results finally for $|\gamma|$ the expression

$$|\gamma| = \left(\frac{2}{\lambda^2} \frac{\cosh \lambda - \cos \lambda}{\cosh \lambda + \cos \lambda} \right)^{\frac{1}{2}} = \left(\frac{1}{\mu^2} \frac{\cosh \sqrt{2}\mu - \cos \sqrt{2}\mu}{\cosh \sqrt{2}\mu + \cos \sqrt{2}\mu} \right)^{\frac{1}{2}} \qquad (13)$$

$$= \varphi_1(\mu),$$

where

$$\lambda^2 = 2(\omega CRl^2) = 2\mu^2$$

because $h(1+i)=ik$, $2ih^2=-k^2$ hence

$$h^2=\frac{-k^2}{2i}=\frac{i\omega CR}{2i} \text{ and } \lambda^2=4h^2l^2=2\omega CRl^2 \text{ and } \mu^2=\omega CRl^2. \quad (14)$$

For large values of λ, γ clearly approaches zero, as $\cosh \lambda_{\lambda=\infty}=\infty$ so that for this case

$$\gamma=\frac{1}{\sqrt{\omega CRl^2}}=\frac{1}{\mu}. \quad (15)$$

For $\lambda=0$ the expression for γ is indeterminate, but its true value for small λ's may be determined by expanding the functions $\cosh \lambda$ and $\cos \lambda$ in series and neglecting the higher powers of λ.

$$\gamma^2=\frac{2}{\lambda^2}\frac{\left(1+\frac{\lambda^2}{2!}+\frac{\lambda^4}{4!}+\right)-\left(1-\frac{\lambda^2}{2!}+\frac{\lambda^4}{4!}-\right)}{\left(1+\frac{\lambda^2}{2!}+\frac{\lambda^4}{4!}\right)+\left(1-\frac{\lambda^2}{2!}+\frac{\lambda^4}{4!}\right)}=\frac{2}{\lambda^2}\frac{\left(\frac{\lambda^2}{2!}+\frac{\lambda^6}{6!}\right)}{\left(1+\frac{\lambda^4}{4!}\right)}$$

$$=\frac{1+\frac{2\lambda^4}{6!}}{1+\frac{\lambda^4}{4!}}=1+\frac{2\lambda^4}{6!}-\frac{\lambda^4}{4!}=1-\frac{28\lambda^4}{6!}=1-\frac{7}{45}\mu^4=1-0.156\mu^4.$$

Finally, extracting the square root

$$|\gamma|=1-\frac{7}{90}\mu^4=1-0.078\mu^4. \quad (16)$$

Thus for $\mu=0$, γ becomes unity, as of course it should. This equation is valid for values of μ less than unity.

In Fig. 5 is shown a plot of this function $\gamma=\varphi_1(\mu)$; φ_1 being explicitly given by equation (13).

Circular Plate Condenser.

In order to obtain a solution for a *circular* plate condenser with an impressed E.M.F. $V_0e^{i\omega t}$ at its rim a similar calculation follows. Instead of equation (11) we shall find a similar one with the trigonometric terms replaced by Bessel's functions.

As the lines of flow on account of symmetry are evidently radial, we may consider the case of an angular *strip* of length ρ cms.,

and 1 cm. wide at the larger end. (Fig. 3). Let R and C denote the re sistance and capacity per unit length respectively, where the strip

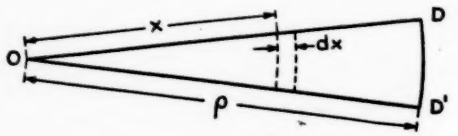

Fig. 3.

is 1 cm. wide. As we go out from the center o the capacity per unit length increases from zero at o to C at the rim $D\,D'$ while the resistance decreases from infinity at o to R at the rim.

The capacity per unit length at any point x of the strip is given by

$$c(x) = \frac{Cx}{\rho}.$$

The resistance is given similarly by

$$r(x) = \frac{R\rho}{x}.$$

Going through a similar demonstration to that for the rectangular plate condenser we find

$$-\frac{\partial I}{\partial x} = \frac{\partial}{\partial t}(dQ) = \frac{\partial}{\partial t}\left(\frac{VCx}{\rho}dx\right) = \frac{Cx}{\rho}\frac{\partial V}{\partial t}dx$$

so that

$$-\frac{\partial I}{\partial x} = \frac{Cx}{\rho}\frac{\partial V}{\partial t} \tag{17}$$

and

$$-\frac{\partial V}{\partial x} = \frac{R\rho}{x}I. \tag{18}$$

Differentiating equation (18) by x and making obvious substitutions there results for the differential equation satisfied by V in this case

$$\frac{\partial^2 V}{\partial x^2} + \frac{1}{x}\frac{\partial V}{\partial x} = CR\frac{\partial V}{\partial t}. \tag{19}$$

The conditions are, that when

$$x = \rho \quad V = V_0 e^{i\omega t},$$

$$x = 0 \quad \frac{\partial V}{\partial x} = 0 \text{ (condition for no current at the center).}$$

Assume that the potential at any point x is given by

$$V = V_0 e^{i\omega t} f(x),$$

where $f(x)$ is to be determined. Substituting this equation into the differential equation we find that $f(x)$ must satisfy the following one:

$$\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial C}{\partial x} + k^2 f = 0, \tag{20}$$

where $k^2 = -i\omega CR$, subject also to the conditions that when

$$\begin{aligned} x &= \rho \quad f(x) = 1, \\ x &= 0 \quad f'(x) = 0. \end{aligned} \tag{21}$$

The solution of (20) is

$$f(x) = AJ_0(kx) + BK_0(kx), \tag{22}$$

where J_0 and K_0 are the Bessels functions of order zero, of the first and second kinds respectively; and where A and B are the two arbitrary constants.

The conditions (21) give

$$\begin{aligned} 1 &= AJ_0(k\rho) + BK_0(k\rho), \\ 0 &= AJ_0'(0) + BK_0'(0), \end{aligned}$$

for the determination of the constants A and B.

As

$$J_0'(x) = \frac{\partial}{\partial x} J_0(x) = -J_1(x)$$

and $J_1(x)$ is zero when x is zero it follows that B is zero and hence

$$A = \frac{1}{J_0(k\rho)}$$

and

$$f(x) = \frac{J_0(kx)}{J_0(k\rho)},$$

so that finally:

$$V = V_0 e^{i\omega t} \frac{J_0(kx)}{J_0(k\rho)}. \tag{23}$$

This equation represents the potential V at any distance x from the center at any time t.

To find the quantity, $Q(t)$ of electricity upon the whole strip at any time we must integrate

$$dQ = VdC = V_0 e^{i\omega t} \frac{J_0(kx)}{J_0(k_i\rho)} \frac{Cx}{\rho} dx,$$

from one end of the strip to the other, obtaining

$$Q(t) = \frac{V_0 e^{i\omega}}{J_0(k_i\rho)} \frac{C}{\rho} \int_0^\rho J_0(kx) \cdot x dx.$$

By means of the integration formula

$$\int_0^x u J_0(u) du = x J_1(x),$$

we obtain for $Q(t)$

$$Q(t) = \frac{V_0 C}{k} \frac{J_1(k_i\rho)}{J_0(k_i\rho)} e^{i\omega t}. \tag{24}$$

We are particularly interested in the maximum quantity of electricity which is contained by the condenser. This quantity is the coefficient of $e^{i\omega t}$ in equation (24).

The quantity of electricity upon the strip, assuming the potential to be V_0 at all points is easily seen to be

$$Q_0 = \int_0^\rho \frac{V_0 C x}{\rho} dx = \frac{V_0 C_i \rho}{2}$$

so that just as in the preceding case we obtain for γ, the ratio of the apparent capacity to its capacity for a steady potential,

$$\gamma = \frac{V_0 C}{k} \frac{J_1(k_i\rho)}{J_0(k_i\rho)} \Big/ \frac{V_0 C_i \rho}{2} = \frac{2}{K\rho} \frac{J_1(k_i\rho)}{J_0(k_i\rho)}. \tag{25}$$

An instructive analogy may be observed between equations (11) and (25). The $J_1(x)$ being an odd alternating or periodic function somewhat similar to a damped harmonic sine function; while $J_0(x)$ is similar to a damped harmonic cosine function. An excellent graphical representation of the two functions $J_0(x)$ and $J_1(x)$ is to be found in Gray and Mathews, Bessel's Functions, page 292. Such analogies are to be looked for, and are a considerable help in proving the probable correctness of results.

It is shown[1] in works on Bessel's functions that

[1] Gray and Mathews, p. 156.

$$\frac{J_1(\eta)}{J_0(\eta)} = i$$

where η is a large imaginary quantity.

If we write

$$k^2\rho^2 = -i\omega CR\rho^2 = -i\mu^2$$

then for large values of $k\rho$ or of μ

$$\gamma = \frac{2i}{\sqrt{-i\mu}} \text{ and its conjugate } \bar{\gamma} = \frac{-2i}{\sqrt{i\mu}}$$

Hence

$$|\gamma| = \frac{2}{\mu} = \frac{2}{\sqrt{\omega CR\rho^2}}. \tag{26}$$

It is instructive to compare this last result with the equation which holds for large values of $\omega CRl^2 = \mu^2$ for the case of a uniform strip,

$$|\gamma| = \frac{1}{\mu} = \frac{1}{\sqrt{\omega CRl^2}}$$

these limiting equations hold for values of $\mu > 4$.

Thus for high frequencies the angular strip is *twice* as good as the uniform one.

That the angular strip would show less diminution in capacity for equal lengths could be seen *a priori*. Consider the two cases represented in Fig. 4. To form an angular strip from a uniform one,

Fig. 4.

we take portion E, which is near the end and difficult for the electricity to reach and place it at F where the current may more easily fill it. The angular strip is thus evidently easier to charge than a corresponding uniform one of equal area and length. It is also easily understood that if the current be made to enter the angular strip at O that it will be theoretically impossible for any

charge to pass through the infinite resistance at the point o. This means practically that when a condenser plate is charged by a wire touching it at its center, that it is very difficult for the charge to spread from, or contract to, the point of contact. For this reason this case would involve far larger discrepancies for the rapid charging and discharging even, than the worst of the two worked out. The problem is theoretically capable of solution only on the assumption that the place where the current enters at o is a small but finite area. Such as the one indicated in the cut. These remarks indicate that such connections should be avoided.

For small frequencies ($\mu < 1$) we may obtain a simple expression for γ by expanding the J_0 and J_1 in series and neglecting all powers of the variable higher than the fourth.

Since

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots$$

$$J_1(x) = \frac{x}{2}\left(1 - \frac{x^2}{2^2 \cdot 2} + \frac{x^4}{2^2 \cdot 4^2 \cdot 3} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2 \cdot 4} + \cdots\right)$$

$$\gamma = \frac{2}{x}\frac{J_1(x)}{J_0(x)} = \frac{1 - \frac{x^2}{8} + \frac{x^4}{192}}{1 - \frac{x^2}{4} + \frac{x^4}{64}} = \frac{\left(1 - \frac{\mu^4}{192}\right) + i\left(\frac{\mu^2}{8}\right)}{\left(1 - \frac{\mu^4}{64}\right) + i\left(\frac{\mu^2}{4}\right)}, \text{ if } x = \sqrt{-i}\mu,$$

$$|\gamma|^2 = \frac{1 - \frac{\mu^4}{96} + \frac{\mu^4}{64}}{1 - \frac{\mu^4}{32} + \frac{\mu^4}{16}} = \frac{1 + \frac{\mu^4}{192}}{1 + \frac{\mu^4}{32}} = 1 - \frac{5}{192}\mu^4.$$

Hence

$$|\gamma| = 1 - \frac{5}{384}\mu^4 = 1 - 0.013\mu^4, \qquad (27)$$

comparing this with the equation (16) for the uniform strip

$$|\gamma| = 1 - 0.078\,\mu^4$$

it is seen that the capacity of the angular strip falls off only about one sixth as fast as that of a uniform one, for very small frequencies.

In order to obtain the points for the intermediate frequencies it is necessary to employ the tables [1] for the values of

$$J_0(u\sqrt{i}) = X - Yi,$$

and were called by Lord Kelevin Ber (u) and Bei (u), meaning the real part and the imaginary part of the Bessels function respectively. The values of

$$J_1(u\sqrt{i}) = -\frac{\partial J_0(u\sqrt{i})}{\partial u} = -\frac{\partial X}{\partial u} + i\frac{\partial Y}{\partial u} = -X' + iY'$$

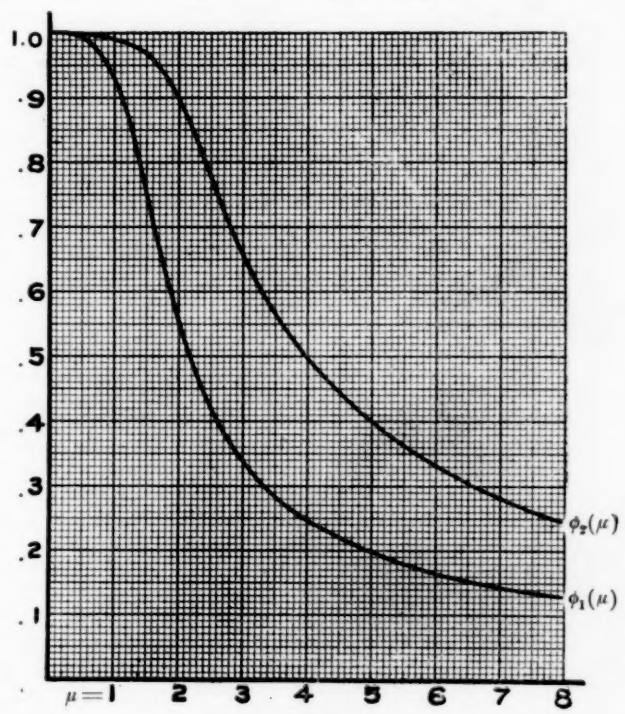

Fig. 5.

The plot shows graphically the two functions

$\gamma = \phi_1(\mu)$ see equation (13) and $\gamma = \phi_2(\mu)$ see equation (28)

for the rectangular and circular condensers respectively.

The ordinate, γ, is the fraction of the ordinarily assumed capacity to be used for any given value of the abscissa, μ.

For the rectangular condenser

$$\mu^2 = \omega CRl^2;$$

for the circular condenser

$$\mu^2 = \omega CR\rho^2$$

where ω, C, R, l and ρ have the same meaning as in the text.

[1] X and Y are tabulated on page 281 of Gray and Mathews.

were deduced from the table by the method of differences. Accordingly, if $\bar{\gamma}$ denotes the conjugate to γ we have

$$\gamma = \frac{2}{\sqrt{-i\mu}} \frac{-X' + iY'}{X - iY}, \quad \bar{\gamma} = \frac{2}{\sqrt{i\mu}} \frac{-X' - iY'}{X + iY},$$

and

$$\gamma\bar{\gamma} = |\gamma|^2 = \frac{4}{\mu^2} \frac{X'^2(\mu) + Y'^2(\mu)}{X^2(\mu) + Y^2(\mu)} = [\varphi_2(\mu)]^2. \tag{28}$$

A plot has been made of the two functions $\gamma = \varphi_1(\mu)$ and $\gamma = \varphi_2(\mu)$ for the two cases represented explicitly by equations (13) and (28) respectively. With any given R, C, l or ρ to every value of $\omega = 2\pi n$ corresponds a definite μ. From the curve the ratio of the apparent capacity (maximum) to the ordinarily assumed capacity (zero frequency) may be easily read off. It is graphically shown in the plot how rapidly the diminution of the acting capacity takes

Table Showing Decrease in Capacity of Condensers with Frequency.

n	μ	μ^2	μ^4	$\gamma = \phi_1(\mu)$	Decrease.	$\gamma = \phi_2(\mu)$	Decrease.
100	—	0.035	0.00122	.999	0.1%	.9998	.02 %
1,000	—	0.35	0.122	.99	1	.999	.1
10,000	1.87	3.5	—	.61	39	.925	7.5
20,000	2.64	7.0	—	.39	61	.75	25.0
40,000	3.74	14.0	—	.27	73	.53	47
Greater.				$= 1/\mu$		$= 2/\mu$	

place, especially after μ is slightly greater than 1.5 and how necessary it is in order to obtain reliable results to assure oneself that the correction is negligible or in case it is not, to carefully apply it.

In order to show how large these corrections may become in a definite case, we shall assume the following values for the constants, C, R, l and ρ. These values are of the order of magnitude that would occur with a half-silvered glass plate condenser, with the plates placed very close together.

$R = 5{,}000$ ohms per cm.

$C = 100$ cms. per cm. $= \frac{100}{9 \times 10^{11}}$ farads per cm.

$l = \rho = 10$ cms.

$\omega = 2\pi n$.

Hence in this case

$$\mu^2 = \omega CRl^2 = \frac{100}{9 \times 10^{11}} \times 5{,}000 \times 100 \times 2\pi n = \frac{35}{10^5} n. \quad (29)$$

The results of this paper in connection with those on "The Effect of Frequency on Self-Inductance"[1] may serve to explain the observed deviations from the equation

$$T = 2\pi\sqrt{LC}$$

in past experimental work of this kind, as it is shown in these papers that *both* L and C tend to diminish in value with increasing frequency. This remark applies all the more forcibly, to work in which high frequencies are employed, such as wireless telegraphy.

Résumé and Conclusions.

The calculations of this paper were undertaken in order to find out how completely a condenser is charged and discharged when the condenser surface has a high resistance such as silvered glass, tin-foil, etc. The calculations are however perfectly general.

In order to attack the problem mathematically it was necessary to assume the simple case of a uniform strip and that of an angular strip, this latter corresponding to a circular plate condenser. The results show that when the surface resistance is high, that the capacity falls off to an enormous extent for comparatively low frequencies. A plot showing the ratio (γ) of the apparent acting capacity to the ordinary capacity has been made for both cases so that for any value of $\mu^2 = \omega CRl^2$ or $\omega CR\rho^2$, we may read off the corresponding values of γ, where C, R, and l or ρ may have any values whatever. The solution for the uniform strip is also one for a cylindrical condenser in which the current enters at one end; as the lines of flow are evidently along the generators. It has been found here that the capacity of a condenser diminishes with increasing frequency as well as the self-inductance of a coil; so that their effects are additive and in all work in which high frequency currents are employed, the experimenter must assure himself that the corrections pointed out are negligible for the work in question, and if not, to apply them.

Physical Laboratory,
College of the City of New York.

[1] The Influence of Frequency upon the Self-inductance of Cylindrical Coils of m-Layers. Physical Review, Vol. XXIII., No. 3, September, 1906, and Bull. Bureau of Standards, Vol. 2, No. 2.

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INFLUENCE OF ATOMIC WEIGHT UPON THE MAXIMA OF ABSORPTION AND REFLECTION BANDS.[1]

BY W. W. COBLENTZ.

FROM various considerations one would expect to find the position of the maxima of the bands of selective absorption and of selective reflection, which are characteristics of certain groups of atoms, to be affected by the molecular weight of the compound. Heretofore the evidence has been more or less contradictory. The difficulty lies in the interpretation of the results. Previous experiments have failed to show a shift of the band with increase in the number groups of atoms in the molecule. This is hardly to be expected if the *cause* of the bands is to be attributed to groups of atoms. On the other hand, the *position* of the band has finally been shown to depend upon the atomic weight of the element (metal) with which group of atoms is combined to form a compound. A recent study of the reflection bands of the sulphates of (H_2, Mg, Ca, Sr, Ba) SO_4, which have a series of reflection bands from 8.2 to 9.4 μ, and of the carbonates of (Mg, Ca, Fe, Cu, Zn, Sr, Ba, Pb) CO_3, which have a complex double band at 6.5 to 7.2 μ, shows conclusively that the maximum of the band shifts toward the long wave-lengths with increase in the atomic weight of the metal, and confirms the writer's previous work on transmission and reflection spectra. In the carbonates the first reflection band shifts from Mg 6.5 μ, Ca 6.58 μ, Fe 6.62 μ, Zn 6.64 μ, Sr 6.72 μ (band not resolved), Ba 6.8 μ, to Pb at 6.94 μ; the second band shifts from Mg 6.86 μ, Ca 6.9 μ; Fe 7.0 μ (?), Zn 6.92 μ, Ba 7.0 μ, to Pb at 7.18 μ.

In the sulphates the first reflection band shifts from Sr at 8.2 μ to Ba at 8.31 μ; the second band shifts from H_2 at 8.62 μ, Mg 8.65 μ, Ca 8.69 μ, Sr 8.78 μ to Ba at 8.87 μ; the third band shifts from K at 9 μ, Sr 9.05 μ, to Ba at 9.11 μ; the fourth band shifts from Sr at 9.28 μ to Ba at 9.35 μ.

In the transmission bands of these sulphates the first band shifts from Mg at 4.53 μ to Ba at 4.62 μ; the second band shifts from Mg at 6.1 μ,

[1] Abstract of a paper presented at the Washington meeting of the Physical Society April 19–20, 1906.

Ca at 6.15 μ to Pb at 6.4 μ; the third band shifts from Ca 6.54 μ to Pb at 6.75 μ, and are additional evidence that the position of the maximum of the band depends upon the atomic weight of the element (metal) with which the group of atoms is combined.

An Accurate Calorimeter.[1]

By Walter P. White.

THIS calorimeter is used in connection with an electric resistance furnace. It is therefore completely enclosed with a double-walled jacket through which water is kept circulating. As the amount of manipulation necessitates the maximum simplicity in methods of measurement, the temperature is determined by a thermo-element, which can be read on a potentiometer almost at the same time with the temperature of the furnace, the differential temperature of the calorimeter and jacket, and the voltage and current used in calibration. The temperatures can be read concordantly to about .001°. The use of large calorimeters and small temperature intervals has been recommended in order to reduce cooling corrections, but at best a small gain in cooling correction, when secured in this way; requires a very large increase in the sensitiveness of the thermometer, while in so far as the cooling errors come from the difficulty of *determining* the cooling correction or from the heat generated in stirring, they are actually greater in the large calorimeter. Hence the temperature interval of 20° was chosen.

Evaporation has been rendered negligible by floating the cover upon the water. The calorimeter resembles in plan a figure eight with a propeller in one side. Dead spaces are thus completely avoided.

Further reasons for a small temperature interval and large calorimeter are found in the closer approximation to Newton's law of cooling and in the reduced effect of the metal parts of the calorimeter. With the present arrangement of jacket and floating cover, however, practically everything within the case is at one of two temperatures, that of the calorimeter water and that of the jacket water, whose difference is accurately measured. Uncertain and lagging temperatures of the metal parts are thus practically eliminated. The slight variation from Newton's law is easily determined and applied as a correction, since the difference in temperature between the calorimeter and its *whole* environment can be at all times exactly known.

Experience has shown that the regularity of the cooling is at least equal to the maximum accuracy obtained in measuring it. The prin-

[1] Abstract of a paper presented at the Washington meeting of the Physical Society April 19–20, 1907.

cipal errors thus far observed have been clearly traceable to lack of sensitiveness in temperature measurement. The selection of a wide temperature interval, therefore, seems completely justified.

Calibrations by the electrical method are still in progress. Seven preliminary results with inferior sensitiveness have already shown an average variation of only two parts in 10,000.

ON THE MEASUREMENT OF THE INTENSITY AND DAMPING OF HERTZIAN WAVES IN AIR.[1]

BY WALTER P. WHITE.

FOR quantitative work in this subject, a thermoelement or bolometer seems necessary. Enclosure in a vacuum is of great assistance, and a fine platinum wire bolometer can be so enclosed much more compactly than a thermoelement. It is also capable of considerably greater sensitiveness than the element. With 30 cm. waves at a distance of 2 m., and a moving coil galvanometer of 2.6 seconds full period, a sensitiveness of 6,000 scale divisions was attained. In spite of this combination of quickness and sensitiveness, the apparatus was very convenient to work with. Working at night, and many other precautions taken by previous observers, were entirely unnecessary.

The bolometer wire was, in the most sensitive arrangement, of platinum, 2.2 μ in diameter, 5 mm. long, with a resistance of 100 ohms. This size of wire was found comparatively easy to handle. The wire was heated 34° by the strongest oscillations. It is mainly owing to the possibility of using finer wire, with a consequent increase in the heating, that the bolometer excels the thermoelement in sensitiveness.

To eliminate variations in intensity a comparison receiver was balanced (by varying its plane of polarization) against the principal receiver, using the galvanometer as a null instrument, and readings agreed to .1 per cent. Very slow variations in the ratio of the two receivers amounting to 2 per cent. sometimes occurred, due possibly to changes in the damping from variations in the spark. Long electrolytic leads between induction coil and oscillator largely eliminated disturbing vibrations and increased the intensity. The induction coil was used on alternating circuit as a transformer, and at least 60 sparks (in kerosene) passed per alternation.

The bolometer lends itself especially well to the production of a sensitive receiver of high damping, which will do much of the work of the long-desired indifferent receiver. By running interference curves and varying the effective intensity of one of the interfering wave trains

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

(by changing its plane of polarization by rotating a grating mirror), the effect of one, two, three, or more waves can be isolated. The law of damping may thus be determined. The highly damped receiver proved sufficiently sensitive for this kind of work, but accurate results were prevented by the oscillations reflected from the iron pipes in the room.

An attempt to use a mercury interrupter for these short oscillations was unsuccessful.

The Effect of the Time of Passage of a Quantity of Electricity on the Throw of a Ballistic Galvanometer.[1]

By F. Wenner.

THE case considered is that of the throw produced when a ballistic galvanometer is connected in series with a condenser and the condenser charged. The galvanometer is of the moving coil type with a radial field and the motion is undamped.

The charging current is represented by the equation,

$$i = Q\frac{a_1 a_2}{a_1 - a_2}\left[e^{-a_1 t} - e^{-a_2 t}\right]$$

where

$$a_1 = \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{CL}} \text{ and } a_2 = \frac{R}{2L} - \sqrt{\frac{R^2}{4L_2} - \frac{1}{CL}}.$$

The moments acting on the coil equated to zero give,

$$I\frac{d^2\varphi}{dt^2} + \beta\varphi = Gi.$$

Substituting the value of the current from the first equation, integrating and making use of well known relations give, when the charge occurs in a time less than the quarter period of the coil and the self inductance is small,

$$Q = \frac{a\,T}{c\,2\pi} \cdot \operatorname{cosec}\left[\tan^{-1}\frac{T}{2\pi CR}\cdot\right]\Phi \cdot \text{ (approximately)}.$$

This equation shows that the time of charge, which depends on the resistance and the capacity, has but little effect unless the product of the resistance and capacity is large or the period of the coil is small.

Experiments with large capacities and variations of the resistance sufficient to change the throw twenty per cent. gave results in fair accord with those calculated from the equation.

The following is the notation used:

I = the moment of inertia of the coil,

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

φ = the angular displacement of the coil,
t = ime,
β = the torsional moment of the suspensions,
G = the coil and field constant,
i = the current during charge,
Q = the charge or quantity,
L = the coefficient of self inductance,
C = the capacity of the condenser,
T = the period of the coil,
a = the steady deflection of the coil,
c = the current producing the steady deflection of the coil,
Φ = the throw of the coil.

A Redetermination of the Wave-Lengths of the Iron Lines.[1]

By A. H. Pfund.

As the work of Michelson and Kayser has shown that not only Rowland's values but Rowland's method of determining wave-lengths are faulty it has been decided by the International Union for Coöperation in Solar Research that a redetermination of the wave-lengths of standard lines be carried out according to the interference method of Fabry and Perot. In order to test this method thoroughly it was agreed upon that several investigators, working independently, carry out determinations on the same (iron) lines, and it was with the object of performing a portion of this work that the present investigation was undertaken.

The method used differs from that of Fabry and Perot only in details: the interferometer plates are kept in a constant temperature enclosure which does not vary by one-twentieth degree centigrade for several weeks; then again the entire Fabry and Perot ring system is projected on the slit of a spectroscope which is so arranged that in its focal plane the spectral lines and the ring system are simultaneously brought to a focus. Each line has superimposed upon it a portion of its ring system and thus results for all of the iron lines are obtainable at a single exposure.

The measurements thus far made agree with those of Fabry and Perot to within less than one part in a million. It is intended to carry out determinations of equal accuracy on the lines of titanium which are far superior, as standards, to the lines of iron.

Johns Hopkins University,
April, 1907.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 19–20, 1907.

Finding Direction by Means of Submarine Sound Signals.

By Lucien Ira Blake.

ONE of the methods at present employed by the Submarine Signal Company of Boston for locating the source of a submarine sound signal is to attach two iron tanks, one to the port — the other to the starboard wall, inside a vessel below the water line and toward the bow. In each of these tanks, filled with water, is suspended a microphonic transmitter which is connected through a port and starboard indicator box to a receiving telephone in the pilot house. According to the ship's course, the different intensities of the sounds received locate the direction of the sound source. In every day practice this is successfully done to within a half of a point, and on about 160 vessels at present, among which are the largest trans-Atlantic steamers. The generally accepted hypothesis that a vessel in the path of a submarine sound casts an acoustic shadow has been used to explain this difference in intensity. It is to be observed, however, that marked differences in intensity occur on the *exposed* side of the vessel, where no shadow exists. The sound is louder when the source is abeam than in any other position. Further, there are decided differences in the *quality* of the sound delivered into the two tanks — even when one is in the alleged shadow. This difference in quality, although with the majority of observers unconsciously perceived, is, I believe, a larger factor in determining sound direction than intensity, and is not explainable on a geometric shadow principle. It would therefore seem necessary to look for some other principle than simple acoustic shadows. So far as variations in intensity are concerned, these can be produced from a cause which is not only amenable to mathematical analysis, but also to experimental demonstration.

From an article by Green, "On the Reflection and Refraction of Sound," Cambridge Transactions, 1838, it is possible to derive two equations, one for the reflected and one for the refracted wave, in terms of a velocity potential. Let φ and φ_1 represent the velocity potentials respectively. Then,

$$\varphi \text{ (reflected)} = \frac{\frac{\rho_1}{\rho} - \frac{\cot\theta_1}{\cot\theta}}{\frac{\rho_1}{\rho} + \frac{\cot\theta_1}{\cot\theta}} \left[\cos\frac{2\pi}{\lambda}\left(-x\cos\theta + y\sin\theta + \nu t\right)\right]^{\frac{1}{2}},$$

$$\varphi \text{ (refracted)} = \frac{2}{\frac{\rho_1}{\rho} + \frac{\cot\theta_1}{\cot\theta}} \left[\cos\frac{2\pi}{\lambda}\left(x\cos\theta_1 + y\sin\theta_1 + \nu_1 t\right)\right]^{\frac{1}{2}},$$

$\rho \cdot \rho_1$ represent densities; θ and θ_1 angles of incidence and refraction.

In the expression for the amplitude of vibration of the refracted sound,

$$\left(\text{viz.,} \ \frac{2}{\frac{\rho_1}{\rho} + \frac{\cot \theta_1}{\cot \theta}} \right),$$

$$\frac{\cot \theta_1}{\cot \theta} = \frac{\nu}{\nu_1} \left[1 - \left(\frac{\nu_1^2}{\nu^2} - 1 \right) tg^2 \theta \right]^{1/2}.$$

Whence the intensity of the refracted sound is proportional to

$$\frac{2}{\frac{\rho_1}{\rho} + \frac{\nu}{\nu_1} \left[1 - \left(\frac{\nu_1^2}{\nu^2} - 1 \right) tg^2 \theta \right]^{1/2}},$$

or depends simply upon the angle θ at which the sound strikes the side of the vessel.

From the above analysis, it is easy to see that, as far as intensity is the factor, direction finding by submarine sound signals must depend upon the angles at which the sound enters the side of a ship — and as (except when the sound is dead ahead or astern) the angles presented by the two sides to the direction of the sound are *different*, the intensities of the two sounds are different in the two receiving tanks and thus the transmitters submerged in these tanks indicate different intensities — the louder on the side where the incident angle is the smaller; that is, on the side toward the sound.

In this analysis it is premised that no sound shadow therefore exists on the remote side of the vessel, but that the sound bends around the vessel as an obstacle, and thus enters the portions of the remote side at angles different from the incident angles of the exposed side. It is easy to calculate how far this assumption may be carried out and what influence it has upon direction finding. It is well known that for long waves impinging upon an obstacle diffraction bands are pronounced and rays of sound for long waves cannot be considered to exist. We can, therefore, calculate how much sound of a given pitch bends around a vessel to enter its remote side, and also the distribution of the diffraction bands on that side. This will lead us to an explanation of the different *quality* of the tones, also of what effect the side of the vessel had and of the effect of the distance of the sound. It gives also a clue as to the best location of the receiving tanks.

A rigid surface, in order to reflect a sound and thereby to produce a shadow back of it, must intercept say at least ten Huyghen's zones. The radius of the nth zone will be obtained from the formula

$$r^2 = \frac{n\lambda}{\frac{1}{R} + \frac{1}{C}}.$$

r = radius of Huyghen's half period zone,
R = radius of curvature of the wave front at the side of the vessel,
C = distance from wave front to the point where the intensity is desired,
λ = wave-lengths,
n = an integer.

It becomes thus a calculation of Huyghen's zones under given conditions of pitch, distance of the sound source and size of the vessel.

I give here for illustration some calculations based upon the pitch of the present standard lightship diaphragm striker of the Submarine Signal Company for a vessel at 5 miles distant from the striker in open sea. The pitch of the bell is $C''' = 1{,}215$ *vs*/sec. and the wave-length is therefore 3.9′ at 60° F., which may be called 4′.

In applying this formula to any particular vessel, one half the breadth of beam is taken as equal to C.

For the Submarine Signal Company's steamer *Sabrina*, $C = 10$ feet approximately. The draft of the steamer is 7 feet.

At 5 miles

$$r_{n=1} = 6.3 \text{ feet.}$$

$$r_{n=10} = 20 \text{ feet.}$$

So that the diffraction bands would be about 2′ apart from the bow with the first one covering 6′. It is therefore clear that maxima and minima of intensity would show from the bow aft, growing weaker and weaker for about 20′— when practically a real shadow would begin.

Taking into consideration, however, the draft of the boat, it is self-evident that the same type of bands of maximum and minimum intensity must start from the keel and extend upward parallel to it to about the same distance of 20′ for the conditions stated. Thus the shadow which the length of the vessel might show would be obliterated by the bending of the sound up around the keel. It is clear, then, that a shadow could not be cast by a vessel for a 4′ wave — unless the vessel was over 40 long and over 20′ draft — and even then the shadow would begin 20′ up from the keel.

The Influence of Frequency on the Resistance and Inductance by Coils.

By Louis Cohen.

IT has been shown experimentally by several investigators that the change in resistance of a coil due to frequency is different from that of a straight wire of the same length as the winding of the coil. The difference arises from the fact that the distribution of the current in the

wire is different in the two cases. Professors Wien, Sommerfeld, and others have independently worked out the problem theoretically and derived certain formulas for the change in resistance. The results, however, obtained by the various investigators are not satisfactory, they do not agree with the experimental results.

In this paper the problem was considered from an entirely different standpoint, and very satisfactory results were obtained, in fact the results arrived at theoretically agree remarkably close with the experimental results obtained by Wien.

According to the theory developed in this paper, the increase in resistance is proportional to the radius of the coil, diameter of wire, pitch, frequency and conductivity of material winding. The final formula obtained is as follows:

$$R = 128 s^2 p^2 \pi d a \sigma \sum_{n=1}^{n=8} \frac{1}{n^2(a_n^2 + \beta_n^2)a_n}$$

where